The Moon as a Test Body for General Relativity

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Summary

Gravity is the force that holds the universe together, yet a theory that unifies it with other areas of physics still eludes us. Testing the very foundation of gravitational theories, like Einstein’s theory of general relativity, is critical in understanding the nature of gravity and how it relates to the rest of the physical world.

Lunar laser ranging (LLR) has been a workhorse for testing general relativity over the past 40 years. The three retroreflector arrays put on the moon by the Apollo astronauts and one of the Soviet Luna arrays continue to be useful targets, and have provided the most stringent tests of the Strong Equivalence Principle and the time variation of Newton’s gravitational constant. However, ground station technology has now reached a point where further advances in range precision, and consequently in the tests of general relativity, will be limited by errors associated with the lunar arrays.

Significant advances in lunar laser ranging will require placing modern retroreflectors and/or active laser ranging systems at new locations on the lunar surface. Ranging to new locations will also enable better measurements of the lunar librations, aiding in our understanding of the interior structure of the moon. More accurate range measurements will allow us to study effects that are too small to be observed by the current capabilities as well as enabling more stringent and crucial probes of gravity.

More advanced retroreflectors are now available. They have the potential to reduce some of the errors associated with using the existing arrays, resulting in more accurate range measurements. Retroreflectors are extremely robust, do not require power, and will last for decades as the Apollo arrays have demonstrated. This longevity is important for studying long-term effects such as a possible time variation in the gravitational constant. Active laser ranging systems, such as asynchronous laser transponders, are also potential options. They additionally have applications for ranging to Mars and other interplanetary bodies with science goals similar to those of lunar ranging.

The principal scientific products of LLR fall into two categories: gravitational science and lunar science. In this white paper, we discuss how the next generation of LLR addresses four key gravitational science questions. In addition, we discuss the current state of retroreflector technology and describe ways in which further advances can be made in both retroreflector and transponder technologies that would enable lighter and more accurate LLR instruments. Lunar science associated with LLR is discussed in another white paper [1].

Key Science Questions:

- Is the Equivalence Principle exact?
- Does the strength of gravity vary with space and time?
- Do extra dimensions or other new physics alter the inverse square law?
- What is the nature of spacetime?
**Introduction**

Over the past 40 years, lunar laser ranging (LLR) from a variety of observatories to retroreflector arrays placed on the lunar surface by the Apollo astronauts and the Soviet Luna missions have dramatically and continually increased our understanding of gravitational physics along with Earth and Moon geophysics, geodesy, and dynamics. We propose to build upon this legacy by starting a new activity to put additional LLR instruments on the moon for advanced gravitational and lunar interior studies. **At present, further technological improvements in the ground-based segment of LLR are rendered futile by the 40-year-old arrays on the moon.**

Installation of retroreflectors was a key part of the Apollo missions, so it is natural to ask if future lunar missions should include them as well. We seek to place hardware on the lunar surface that will sustain LLR progress for decades to come and support future upgrades to the ground stations. A distributed array of state-of-the-art solid retroreflectors is baselined, but we also discuss the possibility of deploying large hollow retroreflectors, laser transponders, and laser communication terminals. The proposed active LLR instruments have the added benefit that they can be adapted for use on Mars or other planetary bodies beyond the moon.

Progress in LLR enabled science is limited by both the properties of the current retroreflector arrays and by their distribution on the lunar surface. The available retroreflectors all lie within 26 degrees latitude of the equator, and the most useful (Apollo) ones within 24 degrees longitude of the sub-earth meridian as shown in Figure 1. This clustering is sub-optimal and weakens their geometrical strength. New retroreflectors placed at locations other than the Apollo sites would enable better measurements of the lunar orientation, impacting the overall value of all range data. In addition, more advanced retroreflectors are now available that will reduce some of the measurement errors associated with using the Apollo arrays [2].

**Figure 1: Location of the lunar retroreflector arrays.** The three Apollo arrays are labeled AP and the two Luna arrays are labeled LUN. LUN1 is unavailable. ORI and SHK show the potential location of two additional sites that would strengthen the geometric coverage and increase the sensitivity to lunar orientation by as much as a factor of four [2].

Table 1 provides a comparative framework for current and future science goals stemming from LLR. In the following sections we summarize the current status of answering four key gravitational science questions and provide some theoretical motivation.

**Is the Equivalence Principle exact?**

The Equivalence Principle (EP), which is based on the equality of gravitational and inertial mass, is a cornerstone of general relativity, putting the theory on a geometric footing. The assumption that
The EP holds in all its forms is built into general relativity, yet efforts to formulate a quantum description of gravity generically introduce new scalar or vector fields that violate the EP [3,4]. Therefore, it is imperative that we subject this apparently exact empirical result to the greatest possible scrutiny.

Two flavors of the EP may be tested via LLR: the weak EP (WEP), and the strong EP (SEP) [5]. The WEP pertains to non-gravitational contributions to mass: namely, Standard Model contributions of nuclear and electromagnetic energy, plus quark masses and their kinetic energies. Nucleons of differing fractional electro-weak and nuclear binding energies might exhibit different couplings to gravity in the case of a WEP violation. The SEP extends the WEP to include gravitational self-energy of a body, addressing the question of how gravity pulls on itself and, therefore, accessing the non-linear aspect of gravity. Laboratory masses lack measurable gravitational self-energy, so bodies of astronomical size must be used to provide sensitivity to SEP violation. LLR provides the best available test of the SEP to date.

A violation of the EP would cause the Earth and Moon to fall at different rates toward the Sun, resulting in a polarization of the lunar orbit. This polarization shows up in LLR as a displacement along the Earth-Sun line with a 29.53 day synodic period. Recent solutions using LLR data yield a WEP test numerically comparable with present laboratory limits, at a part in $10^{-13}$ [6,7]. Combining the laboratory and LLR results together, the possibility of a conspiratorial cancellation is excluded, leaving a rigorous test of the SEP: 

$$\Delta(M_G/M_I)_{\text{SEP}} = (-2.0\pm2.0)\times10^{-13}$$

Because Earth’s self-energy contributes $4.5\times10^{-10}$ of its total mass, this translates to a SEP test of $\sim0.04\%$. Millimeter precision ranges to the moon should deliver order-of-magnitude improvements in EP tests [8].

**Does the strength of gravity vary with space and time?**

Modern attempts to provide a quantum-mechanical framework for gravity often invoke extra dimensions that may be compactified at the Planck scale of $10^{-35}$m.
New scalar fields—for instance represented by the dilaton and moduli that regulate these dimensions—typically evolve over cosmological time scales and produce secular variations in the fundamental “constants.” The same may be said of a residual field left over from inflation, which might be responsible for the apparent acceleration of the expansion of the universe seen today. Thus the gravitational constant, $G$, the fine-structure constant, and the electron-to-proton mass ratio may evolve with time or vary spatially.

LLR is sensitive to variations in $G$. A changing $G$ affects both the monthly lunar orbit and the annual Earth-Moon orbit. For the lunar orbit, changing $G$ and tidal-friction both change the semimajor axis and the orbital period, but with different proportions. Solar perturbations on the lunar orbit are large. Secular change in the annual orbital period from changing $G$ accumulates as an orbital longitude perturbation evolving quadratically with time, $t$. The $t^2$ effect on the phase of the solar perturbations provides a strong limit when measured over decades. Currently, LLR provides the best limits: $\dot{G}/G = (4 \pm 9) \times 10^{-13}$/year [6], which translates to less than 1% variation over the 13.7 billion year age of the universe. Anticipated improvements in LLR data quality, volume, and longevity will accelerate our search for new physics observed via temporal variations in the fundamental constants of nature.

**Do extra dimensions or other new physics alter the inverse square law?**

The inverse square law (ISL) of gravity has been meaningfully tested over length scales spanning 20 orders of magnitude, eliminating Yukawa-like couplings competitive with the strength of gravity from $10^{-4}$ to $10^{16}$ meter length scales. The deepest probe of the ISL is from LLR at a scale of $\sim 10^8$ meters, where any new force must be weaker than gravity by more than ten orders-of-magnitude [9]. Short-range tests of the ISL have recently been prompted by the energy scale of the cosmological acceleration, suggesting a new-physics length scale at $\sim 0.1$ mm [10].

On cosmological scales, brane-inspired modifications to gravity (such as DGP gravity) attempt to account for the apparent acceleration of the universe, and in so doing produce influences on the lunar orbit within a factor of about 10 of current measurements [11,12]. Finally, covariant versions of the “modified Newtonian dynamics” (MOND) paradigm, such as TeVeS [13], may be subject to clear falsification by precision ranging measurements within the solar system. Current LLR capabilities fall short of testing these ideas, but not insurmountably. Improved equipment on both the Moon and Earth can conceivably cover another two orders of magnitude in the coming decade opening up the possibility for a major discovery.

**What is the nature of spacetime?**

The recent and unexpected measurement of the accelerating expansion of the universe has provided new motivation for exploring the nature of spacetime. Models that predict modification of gravity at large distances, such as brane-world models, have recently become of interest [14]. These theories exhibit a strong coupling phenomenon that makes the gravitational force source dependent. These theories become testable at shorter distances where the coupling sets in for lighter sources [15].
The Earth-Moon system provides a testbed for investigating the nature of spacetime at solar-system scales. For example, general relativity predicts that a gyroscope moving through curved spacetime will precess with respect to a rest frame. This is referred to as geodetic or de Sitter precession. The Earth-Moon system behaves as a gyroscope with a predicted geodetic precession of 19.2 msec/year. This is observed using LLR by measuring the lunar perigee precession.

The current limit on the deviation of the geodetic procession from the general relativity prediction is: \( K_{gp} = (-1.9 \pm 6.4) \times 10^{-3} \) [6]. This measurement can also be used to set a limit on a possible cosmological constant: \( \Lambda < 10^{-26} \text{ km}^2 \) [16], which has implications for our understanding of dark energy.

It is also useful to look at violations of general relativity in the context of metric theories of gravity. Parameterized Post-Newtonian (PPN) formalism provides a convenient way to describe a class of deviations from general relativity. The most often considered PPN parameters are \( \gamma \) and \( \beta \): \( \gamma \) indicates how much spacetime curvature is produced per unit mass, while \( \beta \) indicates how nonlinear gravity is (self-interaction). \( \gamma \) and \( \beta \) are identically one in general relativity. Limits on \( \gamma \) can be set from geodetic precession measurements [17], but the best limits presently come from measurements of the gravitational time delay of light, i.e. the Shapiro effect. Doppler measurements to the Cassini spacecraft set the current limit on \( \gamma \): \( (\gamma - 1) = (2.1 \pm 2.3) \times 10^{-5} \) [18]. This combined with LLR SEP results provides the best limit on \( \beta \): \( (\beta - 1) = (1.2 \pm 1.1) \times 10^{-4} \) [6]. Scalar tensor theories with “attractors” for the cosmic background scalar-field dynamics predict a residual \( \gamma - 1 \) and perhaps \( \beta - 1 \) of order \( 10^{-7} - 10^{-5} \) today [3], within reach of advanced LLR and spacecraft time-delay measurements.

**Next Generation LLR**

Five retroreflector arrays were placed on the Moon in the period 1969–1973. US astronauts placed three during the Apollo missions (11, 14, and 15), and two French arrays were sent on Russian Lunokhod rovers. All the Apollo arrays and the array on Lunokhod 2 are still viable targets today.

![Figure 2: The Apollo 15 retroreflector array was made from 300 fused silica cubes. The physical size of the array is now limiting ranging measurements.](image)

The first LLR measurements had a precision of about 20 cm. Since 1969, several stations have successfully ranged to the lunar retroreflectors and have increased the range accuracy by a factor of 10 to the level of a few centimeters. Poor detection rates have historically limited LLR (not every laser pulse sent to the Moon results in a detected return photon). However, the relatively new APOLLO system uses the large collecting area of the Apache Point telescope and has very efficient avalanche photodiode arrays such that thousands of detections are recorded (even multiple detections per pulse) leading to a statistical uncertainty of about 1 mm for timescales of less than 10 minutes.
The dominant random uncertainty per photon received by modern LLR stations stems from the size and changing orientation of the reflector array due to the lunar librations and the associated spread of pulse return times. Additionally, at the millimeter level of precision, systematic errors associated with lunar arrays (such as the thermal expansion of the array support structure and underlying regolith) start to become significant. Progressing beyond this level of precision will require new lunar retroreflectors or laser transponders designed to be thermally stable and to reduce or eliminate orientation-dependent pulse spreading.

![Image](image.jpg)

**Figure 3:** Improvements in the ground station technology over the past 40 years have increased the range accuracy by 2 orders of magnitude. Errors associated with the existing retroreflectors are now becoming a limitation.

Large single cube corners (>10 cm) can potentially be made to provide similar return rates as the Apollo arrays, without significant pulse spreading. To further increase the total response, several cubes could be deployed around a landing site with enough separation that responses seen by the Earth stations do not overlap.

Solid cube corner retroreflectors (up to 11 cm) have flown on over a hundred missions, for both satellite and lunar arrays. The proposed large cube pushes the state of the art, but recent tests of a 10 cm cube have demonstrated it meets relevant requirements for the lunar environment. Designs for the housing are still in development and its qualification is making good progress [19].

![Image](image.jpg)

**Figure 4:** A 10 cm solid cube corner reflector was recently qualified for the lunar environment. Also shown for comparison is a 3.8 cm Apollo engineering model cube corner.

Hollow cube corners are also a promising alternative to the traditional solid cubes. They potentially weigh less, have smaller thermal distortions, do not suffer from thermal changes in their index of refraction, and do not introduce significant polarization effects. Thus, they can be made larger without sacrificing as much in optical performance. Hollow cubes have flown on a few missions, but are generally not used on satellites for laser ranging because of a lack of test data and some indications of instabilities at high temperatures. Advances in adhesives and other techniques for bonding hollow cubes makes it worthwhile to further develop this technology for lunar applications.

Isolation from ground motion and thermal changes are also key for going beyond the Apollo array capabilities. Each reflector should be rigidly grounded to directly sense lunar body motion and be located far enough away from normal human
activity to avoid vibration and contamination (dust) from affecting the cubes. Advanced retroreflectors would also benefit from being thermally coupled to the ground below the surface layer. This would require drilling a hole about a meter deep and inserting a thermally stable rod (high conductivity, low coefficient of thermal expansion). The retroreflector would then be mounted to the exposed end of the rod. A thermal blanket positioned over the lunar surface around and below the retroreflector would also be of benefit.

Active laser transponders are a promising alternative to passive retroreflectors [20]. Active transponders are devices that both send and receive predictable signals and can be used for ranging and time transfer. Laser transponders have approximately a R² link advantage over direct ranging loss of 1/R⁴, essentially because the signal is propagating in only one direction before being regenerated. **With the development and inclusion of laser communications for spaceflight missions, it is logical to include an optical transponder that uses the same opto-mechanical infrastructure with minimal impact on the mission resources.** These instruments could be used to support the proposed science in addition to providing communications support to the astronauts and/or other scientific instruments. These lunar instruments would also provide a pathfinder for applications on Mars and other planetary bodies.

While both the retroreflector and transponder options would benefit from further development, existing technology is sufficient to support near-term opportunities such as the International Lunar Network (ILN) [21]. Putting a single large cube corner on the ILN would immediately provide better geometric coverage, thus increasing the overall measurement sensitivity to the lunar motion. This opportunity could serve as a pathfinder for more sophisticated emplacement by the astronauts, enabling unprecedented tests of our fundamental understanding of gravity.

**References**