Astronomy 2020 – Space Astronomy & Exploration **Fall 2020**
Homework #2 Due: Sep. 16, 2020

In questions 1-3 below, choose the best answer. Then explain your reasoning in a few complete sentences. Why is your answer correct?

1. (2 pts). Artificial gravity equivalent to 1g can be achieved aboard a spacecraft by
   a. Firing the engines continuously to produce a constant acceleration of 1 meter/sec^2.
   b. Doing an initial engine burn to achieve a velocity of 0.9c and coasting to your destination at this speed according to Newton's first law.
   c. Rotating the spaceship with a centripetal acceleration that equals the acceleration of gravity on Earth's surface.
   d. All of the above.

An acceleration, either linear or centripetal, equivalent to 1 g (approximately 10 meter/sec^2), which is the acceleration of gravity on Earth, is required to produce artificial gravity equal to that on Earth.

2. (2 pts). The mass of Jupiter can be calculated by
   a. measuring the orbital period and distance of Jupiter's orbit around the Sun.
   b. measuring the orbital period and distance of one of Jupiter's moons from Jupiter.
   c. measuring the orbital speed of one of Jupiter's moons.
   d. knowing the Sun's mass and measuring how Jupiter's speed changes during its elliptical orbit around the Sun.
   e. knowing the Sun's mass and measuring the average distance of Jupiter from the Sun.

Newton's version of Kepler's 3rd Law, which we derived in class, relates orbital period and orbital radius to the mass. This applies to planets going around the Sun and moons going around planets.

3. (2pts). When NASA's *Voyager 2* passed by Saturn, its speed increased (but not due to firing its engines). What must have happened?
   a. *Voyager 2* must have dipped through Saturn's atmosphere.
   b. Saturn's rotation must have sped up slightly.
   c. Saturn must have lost a very tiny bit of its orbital energy.
   d. Saturn must have captured an asteroid at precisely the moment that *Voyager 2* passed by.

During the Voyager fly-by, the spacecraft received a "gravitational assist" or "gravitational slingshot" to boost the velocity of Voyager. This can be understood via conservation of energy and/or angular momentum in which Saturn transfers a bit of orbital energy to Voyager while the total energy remains constant.

4. (6 pts). The science fiction author, Arthur C. Clarke (*2001: A Space Odyssey*) was the first to conceive the idea that a satellite could be placed into geostationary orbit. A geostationary orbit is a circular orbit that lies above the Earth's equator, follows the direction of Earth's rotation, and has an orbital period that equals the Earth's rotational period (one day).

a. Using Newton's 2nd Law of Motion, the Law of Gravitation, the formula for centripetal acceleration, and the relationship between orbital velocity and period, derive an equation that relates the orbital period to the orbital radius.

Newton's 2nd law tells us that the force, F, causing the orbit of the satellite is related to the satellite's acceleration, a, through $F = m_{sat}a$. Since acceleration in a circular orbit (centripetal acceleration) is $a = {v^2}/{r}$, the force must be $F = {m_{sat}v^2}/{r}$. Since the force is supplied by gravity, it is also equal to $F = \frac{GM_{Earth}m_{sat}}{r^2}$. Setting the two expressions for the force equal and cancelling $\frac{m_{sat}}{r}$ gives us $\frac{GM_{Earth}}{r} = v^2$. Since $v = \frac{2\pi r}{T}$, this means $\frac{GM_{Earth}}{r} = \frac{4\pi^2 r^2}{T^2}$. With some rearranging, this is $r = \sqrt[3]{\frac{GM_{Earth}T^2}{4\pi^2}}$.

b. Using this equation, calculate orbital radius of a satellite in geostationary orbit. How high is this above the Earth's surface in kilometers?

Using $r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$, $G = 6.67 \times 10^{-11}\ \frac{m^3}{kg\ s^2}$, $M_{Earth} = 5.97 \times 10^{24}\ kg$, and $T = 1\ day = 86400\ s$, we find an orbital radius of $r = 4.22 \times 10^7\ m$. r is the distance from the center of the Earth to the satellite. The distance from the surface of the Earth to the satellite is $r - R_{Earth} = 4.22 \times 10^7\ m - 6.4 \times 10^6\ m \approx 3.58 \times 10^7\ m = 35{,}800\ km$.

c. How does this altitude for a geostationary orbit compare to that for a satellite in Low Earth Orbit (LEO)?

Geostationary orbits, at $\approx$ 36,000 km from the Earth's surface have an altitude about 20 times greater than LEO, which is defined by an altitude of $<$ 2,000 km.

5. (2 pts). Give an example in which thermal energy might be converted to gravitational energy.

One example is: A hot air balloon rises: as we heat the gas in a balloon, the internal pressure increases and the balloon expands. Therefore, the density of the air inside decreases and when the average density of the entire balloon (balloon material plus basket plus air inside) becomes less than the density of air outside, the balloon rises, gaining gravitational energy.

6. (4 pts). Orbits.

a. Suppose the Sun were replaced by a star with twice as much mass. Could Earth's orbit stay the same? Why or why not?

If the Sun were replaced by a star with twice the mass, the Earth would feel a force twice as large with no change to its inertia; so, it would feel a doubled acceleration causing its orbit to change.

b. Suppose Earth doubled in mass but the Sun stayed the same as it is now. Could Earth's orbit stay the same. Why or why not?

If the Earth doubled in mass, it would feel the same acceleration (because the gravitational force it feels doubles, but its inertia doubles as well); so, its orbit could remain unchanged.

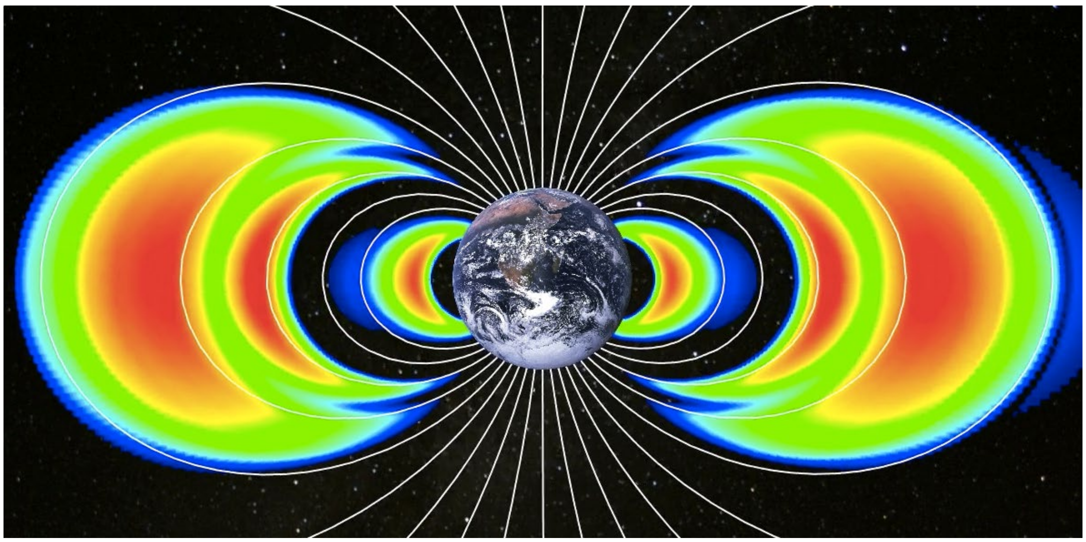
7. (5 pts). Explorer 1 discovered the Van Allen radiation belts as shown below (color represents intensity of radiation).

a. Explain how these radiation belts are related to the Earth's magnetosphere?

High energy particles from the solar wind become trapped inside the Earth's magnetic field, thus effectively lighting up the magnetic field by these particles as seen first by Explorer1 and also by the recent Van Allen probes

b. How does this magnetic field protect us from harmful space radiation?

The belts trap solar wind and the magnetic field deflects energized particles from space, protecting us from exposure to radiation. Without a magnetic field, this harmful radiation would penetrate to the Earth's surface, endangering life and electronic equipment.



8. (4 pts). The Moon orbits Earth in an average of 27.3 days at an average distance of 384,000 kilometers. Using Newton's version of Kepler's third law that we derived as a class exercise, determine the mass of Earth.

Using the Moon's orbital period and distance, the mass of Earth is about 6.0×10^{24} kg.

$$M_{\text{Earth}} \approx \frac{4\pi^2}{G} \frac{(a_{\text{Moon}})^3}{(P_{\text{Moon}})^2}$$

Making sure that we use appropriate units, we find:

$$M_{\text{Earth}} \approx \frac{4\pi^2 \left(384{,}000 \text{ km} \times 1{,}000 \frac{\text{m}}{\text{km}}\right)^3}{\left(6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \times \text{s}^2}\right)\left(27.3 \text{ days} \times 24 \frac{\text{hr}}{\text{day}} \times 3{,}600 \frac{\text{s}}{\text{hr}}\right)^2}$$

$= 6.0 \times 10^{24}$ kg.

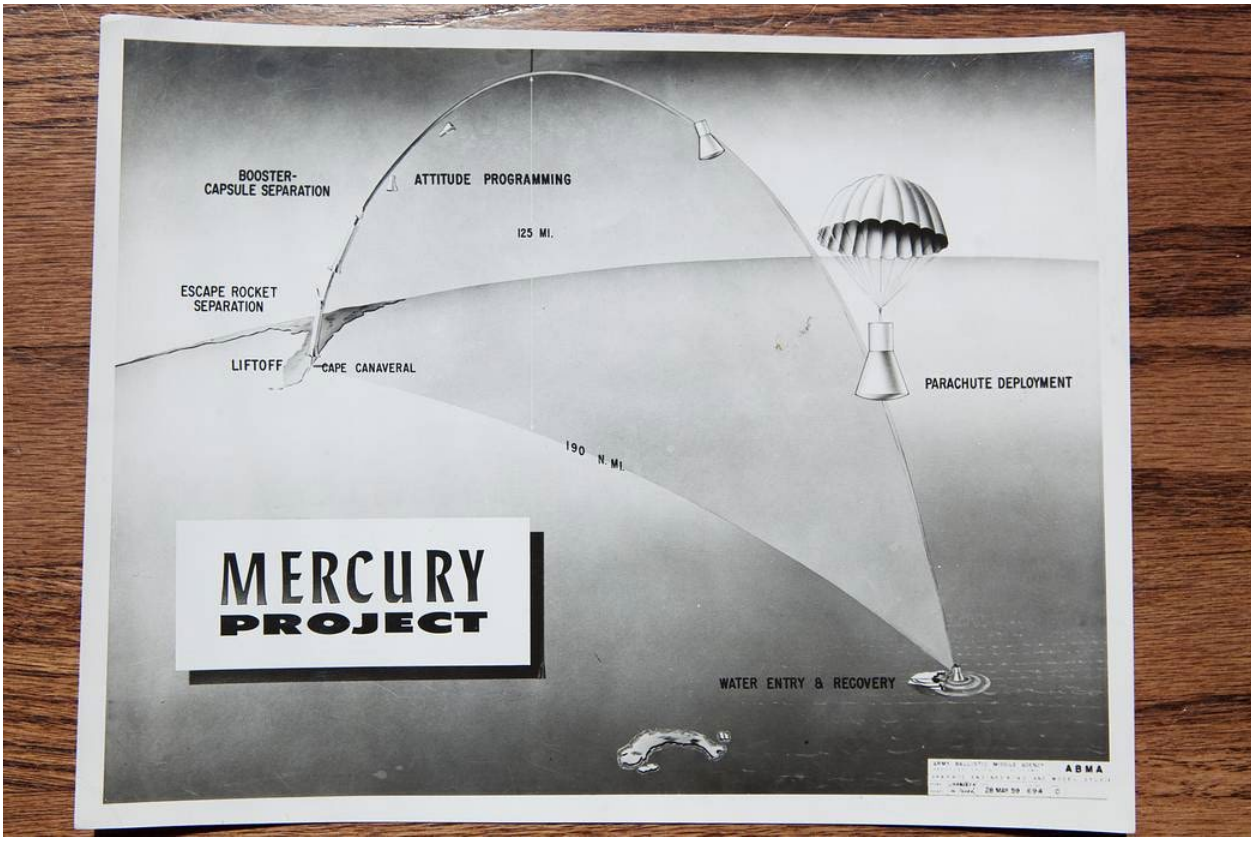
9. (2 pts). Explain (a) why orbits cannot change spontaneously and (b) how a gravitational encounter can cause a change in orbit.

(a) Orbits cannot change spontaneously because their energies and angular momenta are fixed since both of these quantities are conserved even in the presence of gravity.

(b) A gravitational encounter can cause a change in orbit because the two bodies in the encounter can exchange energy and angular momentum since only the grand totals of these quantities is conserved.

10. (2 pts). In 1961, Alan Shepard flew the first suborbital mission for NASA as part of Project Mercury. What was the trajectory for his Redstone rocket? Discuss what would have been required for Shepard to achieve orbit around the Earth as was done later by John Glenn.

Here is a historical picture of Shepard's trajectory as posted on NASA's website. It is a parabola reaching a suborbital altitude of 125 miles and downrange distance of 190 miles to splashdown in the ocean. An interesting side note is that NASA research mathematician Katherine Johnson (portrayed in the *Hidden Figures* movie) did trajectory analysis for the mission.



To achieve orbit, the launch velocity needs to be higher and this required a larger engine and rocket used by John Glenn. The rocket equation allows us to calculate the amount of fuel needed to achieve orbit given the exhaust velocity of the rocket and orbital velocity.