Testing Gravitational Theory by Accurate Ranging To Mercury

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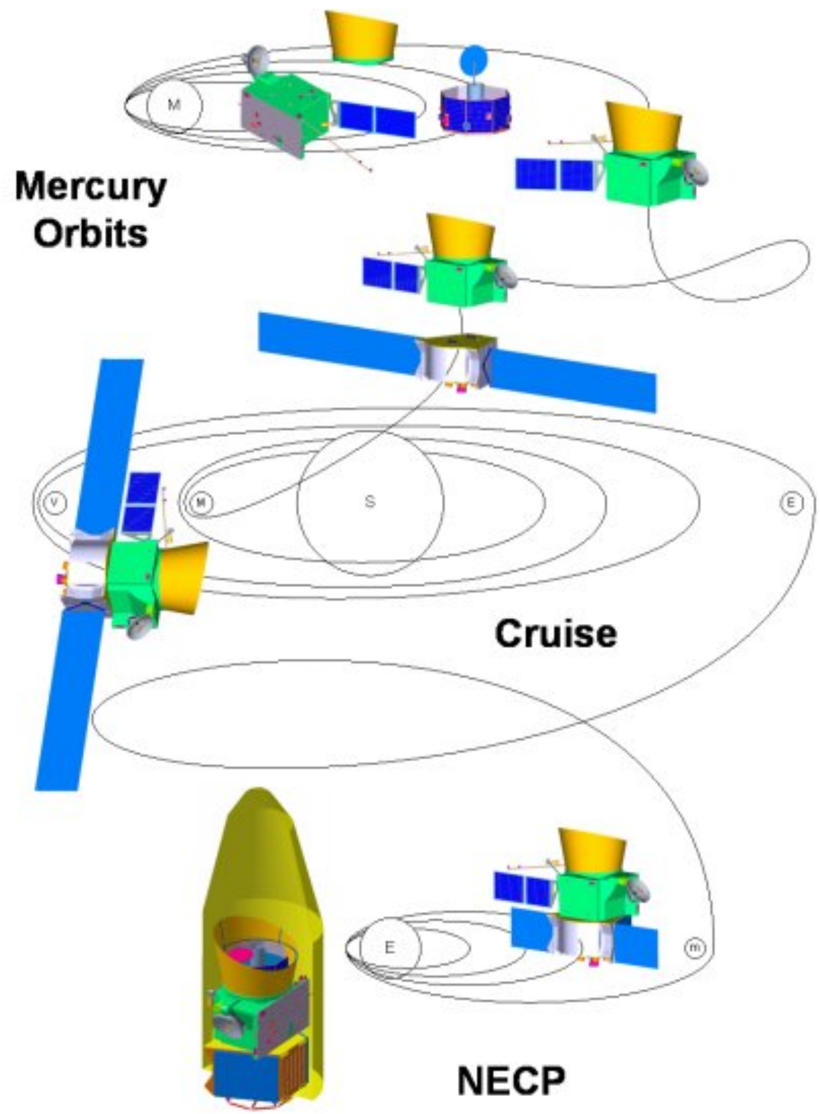
+ Graduate Students
+ Post-Docs

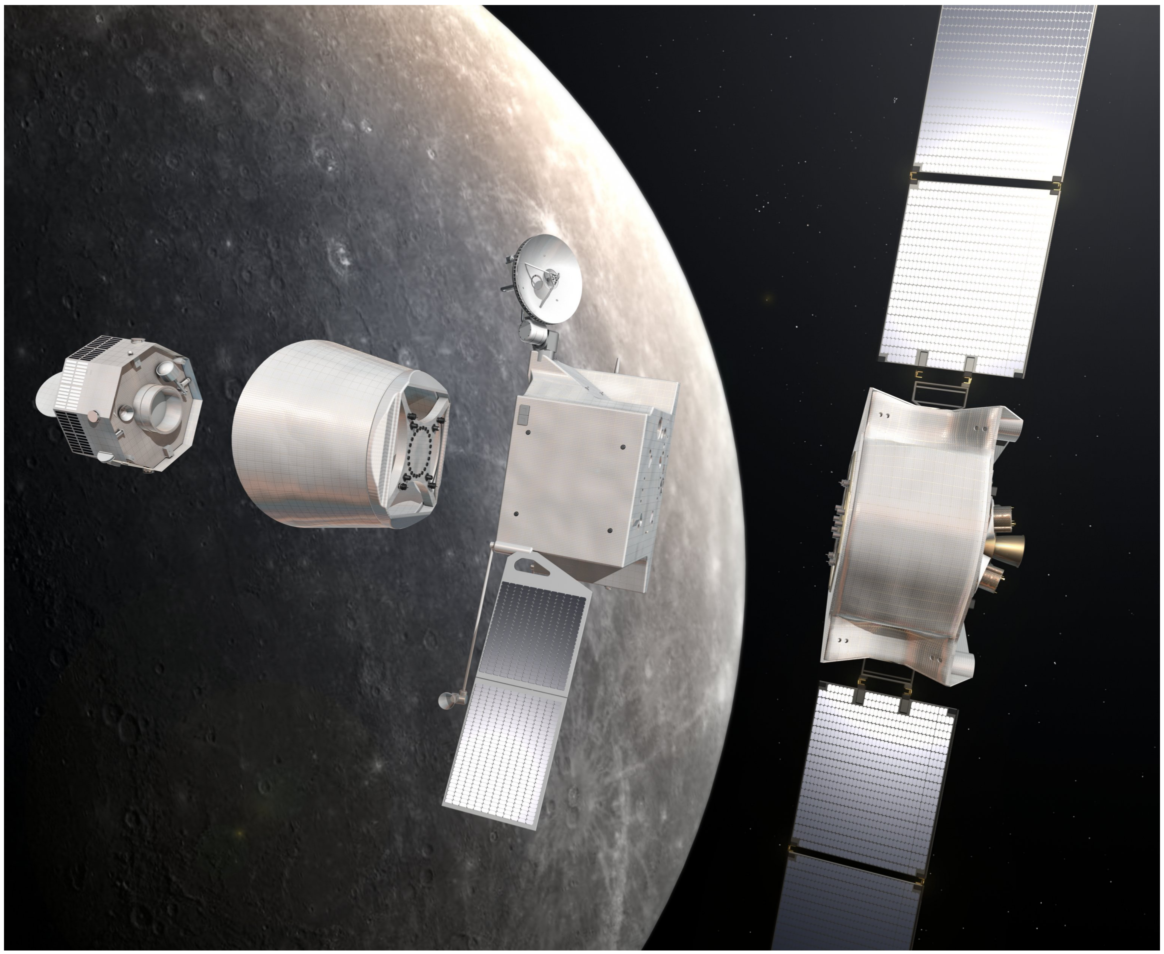
History:

~1975 simulation studies begun
1980-1982 NASA funding
1989-1995 some publications
~1995 ESA studies on BepiColombo
2004 MORE funded by ESA



BepiColombo: ESA/JAXA Cornerstone Mission to Mercury

MISSION CONTROL: ESOC, Darmstadt



LAUNCH: Fregat-Soyuz 2-1B in 2014; two gravity assists at Venus, plus two assists at Mercury

PROPULSION: solar electric for 4.2 years, followed by chemical propulsion for Mercury capture and orbit insertion (2020).

Ranging Instrumentation

Radio Science Instrument is one of about 15 instruments on the Planetary Orbiter

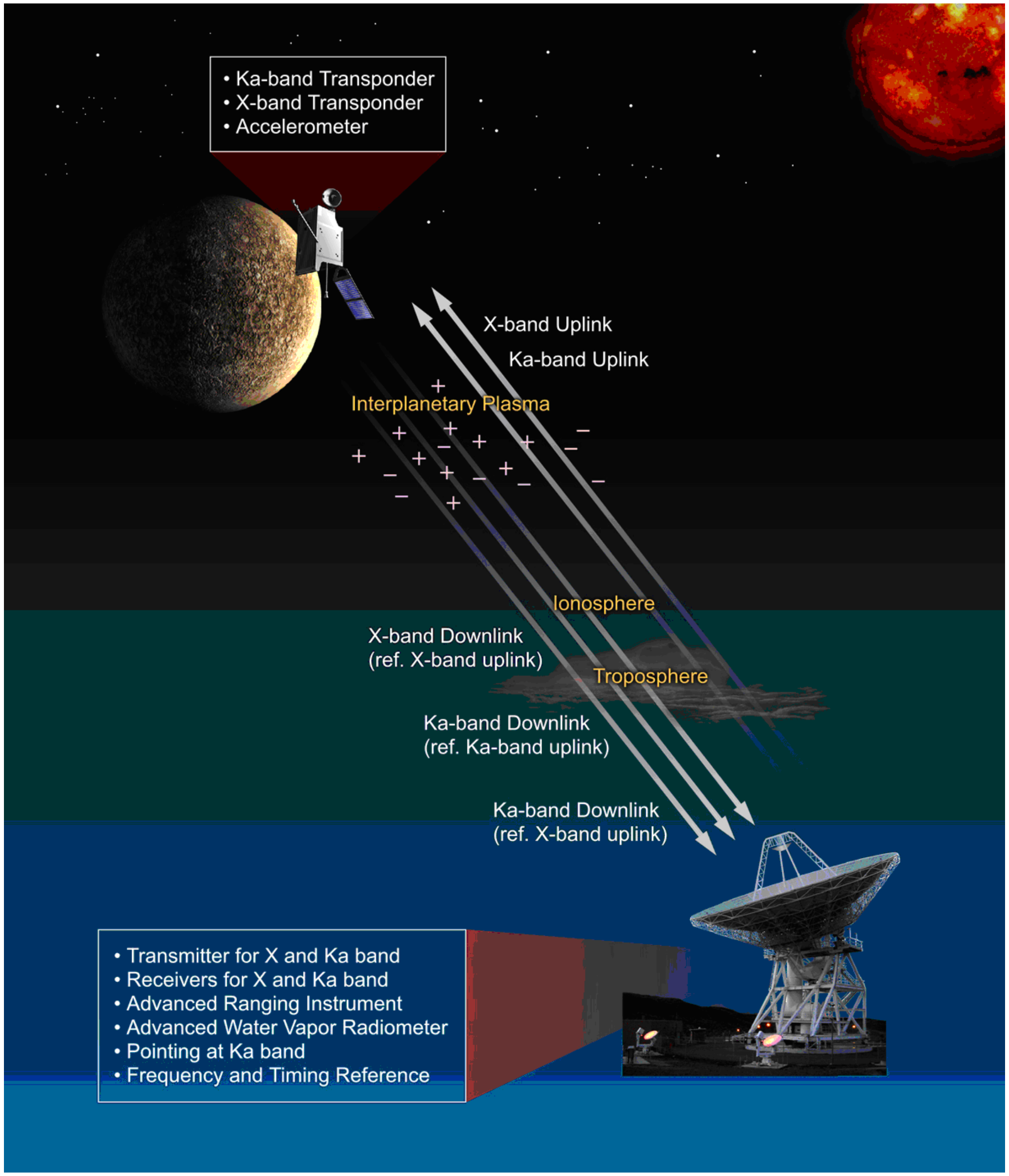
Its purpose is verification of the Theory of Relativity:

"MORE" –Mercury Orbiter
Relativity Experiment

Ranging Accuracy:

1 normal point per day with average range uncertainty during the mission:

$$\sigma = 4.50\,\mathrm{cm}$$



Covariance Analysis &
Modified Worst Case Systematic Error

The observable is the one-way range between earth & mercury:

$$\rho_i = \rho(t_i, d_1, d_2, \ldots d_m).$$

Error analysis involves the covariance matrix and its inverse:

$$C_{mn} = \sum_i \frac{\partial \rho_i}{\partial d_m} \frac{\partial \rho_i}{\partial d_n}; \;\; B_{mn} = \left(C^{-1}\right)_{mn}.$$

For random uncorrelated measurement errors: $\delta d_m = \sigma \sqrt{B_{mm}};$

$\rightarrow N^{-1/2} \text{ as } N \rightarrow \infty$

For some chosen parameter d_m what is the largest possible error that could arise from systematic errors that are correlated with effects due to other parameters?

$$\delta d_m = \sigma \sqrt{N B_{mm}};$$

N is the number of measurements.

$\rightarrow \text{const. as } N \rightarrow \infty.$

Solar System & Relativity Parameters

Cosmological parameter: $\dot{G}/G$

Solar Parameters: $GM_{\odot}, J_2$

Orbital Parameters: 12 – 3 = 9 (due to rotational invariance of range)

Relativity Parameters:

γ, spatial curvature;

β, nonlinear contribution to g_{00};

$\alpha_1, \alpha_2, \alpha_3$, preferred frame parameters;

ξ, Whitehead theory-galactic interaction;

η, Nordvedt parameter

--violation of strong equivalence principle

Total: 9+10=19 (α_3 is currently left out)

Simple Example—perturbations due to β

$$S_{mn} = \int_0^t (\sin f)^m (\cos f)^n \, df; \; S_{00} = f - f_0;$$

$$S_{10} = -\cos f + \cos f_0; \;\; S_{01} = \sin f - \sin f_0;$$

$$S_{11} = \frac{1}{2}((\sin f)^2 - (\sin f_0)^2);$$

$$\Delta a = 2\frac{GM_\odot}{c^2}[2S_{10} + 2eS_{11}]/(1-e^2)^2; \; \Delta e = \Delta a(1-e^2)/(2ea);$$

$$\Delta\Omega = \Delta I = 0;$$

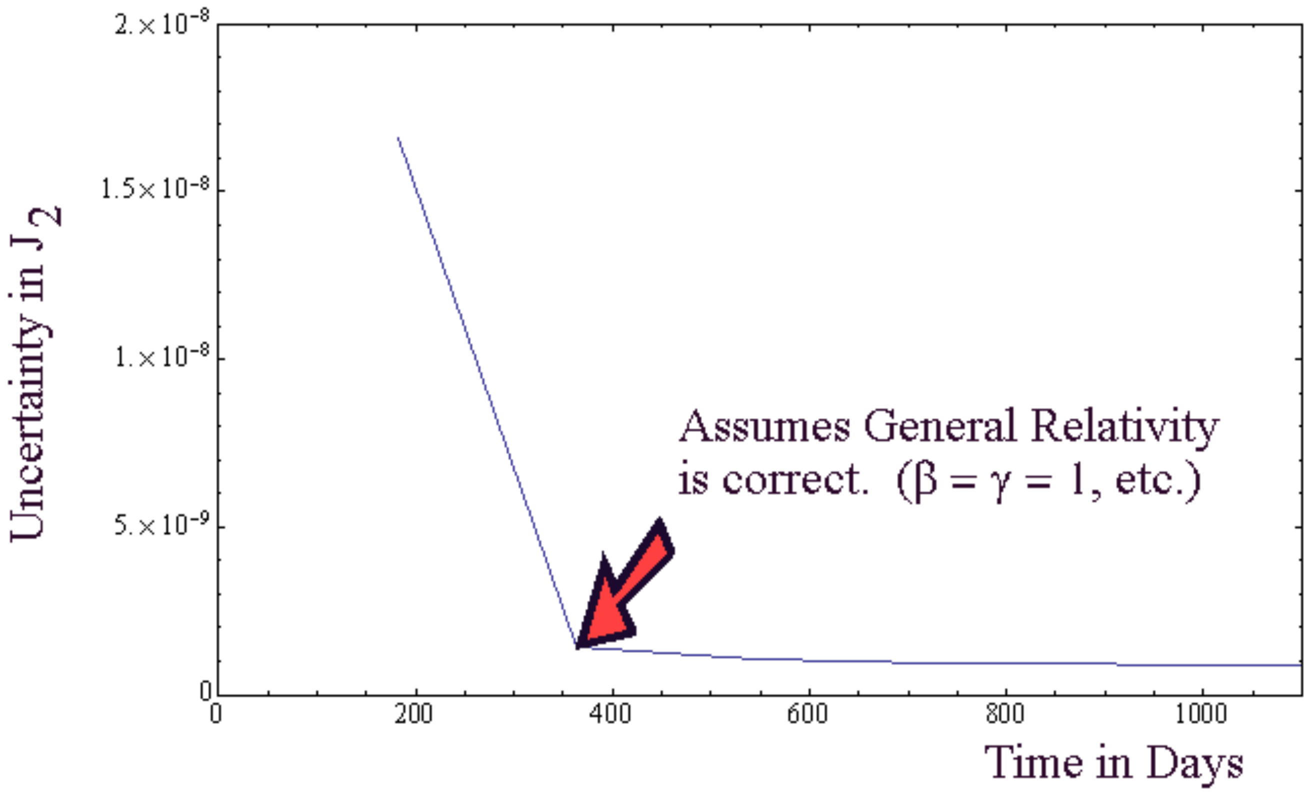
$$\Delta\tilde{\omega} = \frac{GM_\odot}{a(1-e^2)c^2}[-S_{00} - 2S_{01}/e - \sin f \cos f + \sin f_0 \cos f_0];$$

$$\Delta M = -3\frac{GM_\odot}{a(1-e^2)^2 c^2}(1+e\cos f_0)^2 \sqrt{\frac{GM_\odot}{a^3}}(t - t_0) +$$

$$\frac{GM_\odot}{a\sqrt{(1-e^2)}c^2}[2S_{01}/e + \sin f \cos f - \sin f_0 \cos f_0]$$

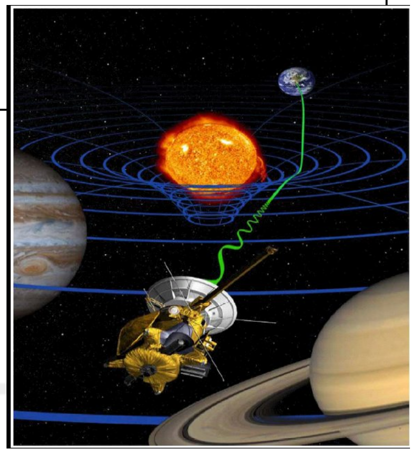
Mean Anomaly M ⟶ Eccentric Anomaly E ⟶ True Anomaly f

Calculation Results—Quadrupole Moment



Current Uncertainty: about 10^{-7}

How well is gamma determined?



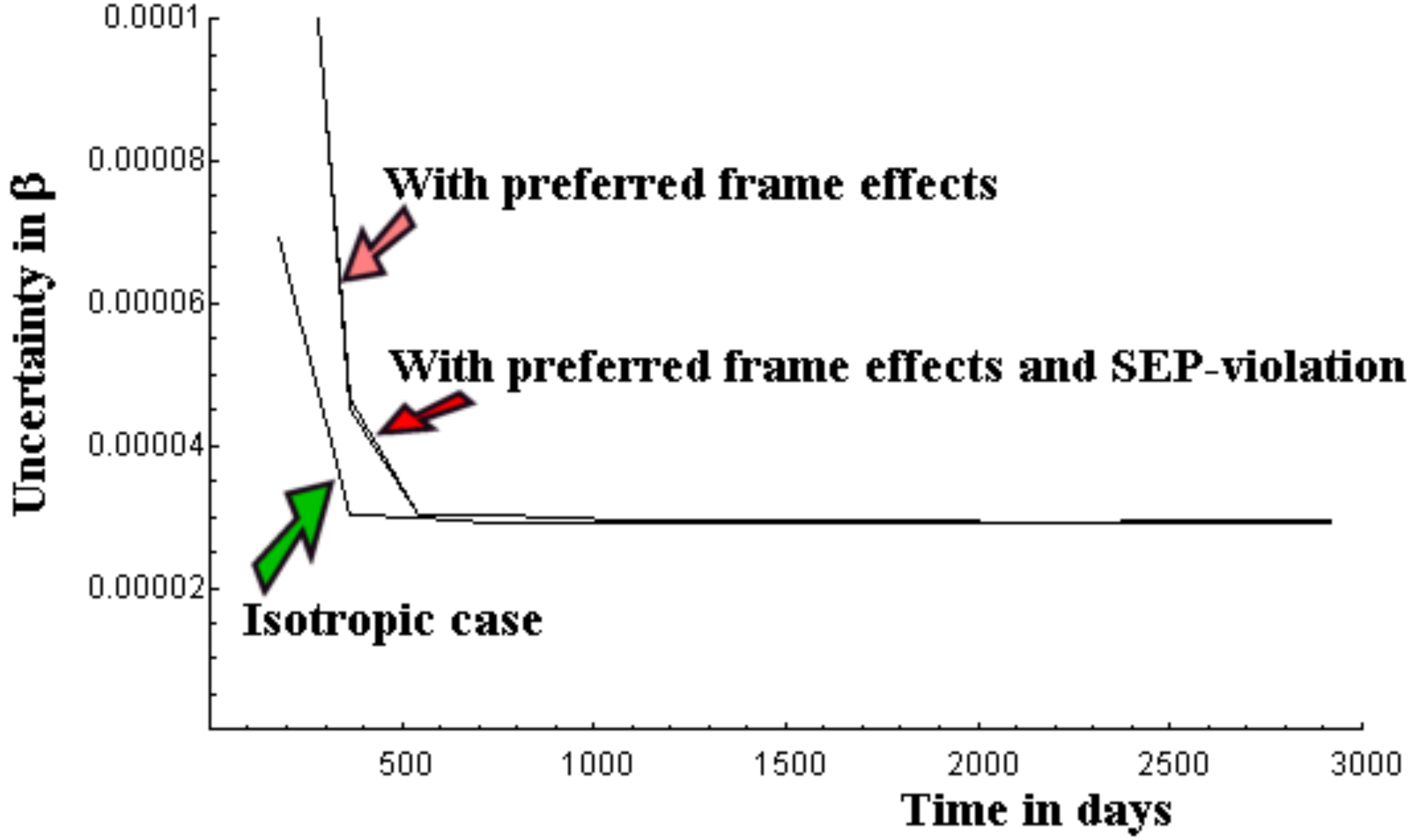
The result

- Cassini-Earth-Sun conjunction has allowed to estimate gamma in agreement with GR (B. Bertotti, L.Iess & P.Tortora, Nature, ***425***, 2003)

$$\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$$

Results for γ

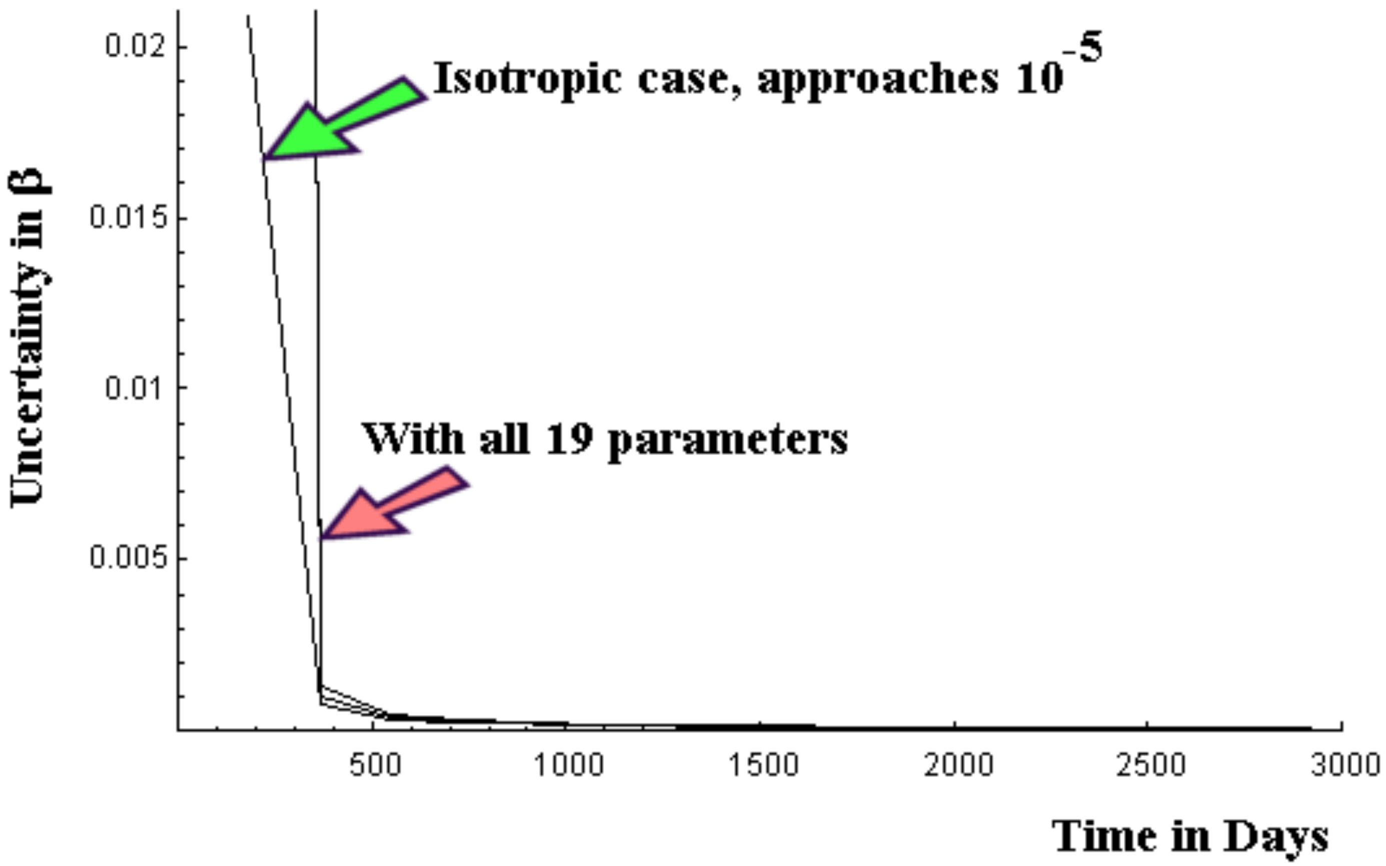
Current uncertainty: .000023



This assumes the line-of-sight is no closer to the sun than 5 degrees.

Results for β

Current uncertainty: .003



Strong Equivalence Principle Violation

Gravitational self-energy, particularly of the sun, causes an anomalous direct planetary perturbation. In particular, the orbits of Earth and Mercury are polarized by the anomalous attraction to Jupiter. In heliocentric coordinates, the anomalous acceleration of planet i toward Jupiter (j) is

$$\frac{d^2\mathbf{r}_i}{dt^2} = \eta(\Omega_\odot - \Omega_i)\frac{GM_j}{\left|\mathbf{r}_i - \mathbf{r}_j\right|^3}\left(\mathbf{r}_i - \mathbf{r}_j\right),$$

$$\Omega_\odot = -3.52\times 10^{-6}; \quad \Omega_i \text{ are negligible}$$

This acceleration cannot be integrated exactly because the distance in the denominator is too complicated. However the expression can be expanded in a convergent series in the parameter $\frac{a_i}{a_j}$

Analysis of SEP-violating accelerations

Assuming as a first approximation that the planet and Jupiter are in the same plane (very good for Earth), the perturbing acceleration can be expressed as a Fourier series in multiples of the synodic orbital frequency

$$\Omega_s = \omega_i - \omega_j$$
$$S_i = (\omega_i - \omega_j)(t - t_0),$$

and then can be separated into coupled differential equations for the tangential and radial parts. These equations can be solved analytically. (See Laplace, *Celestial Mechanics* (about 1830) about how to do this.)

Earth:

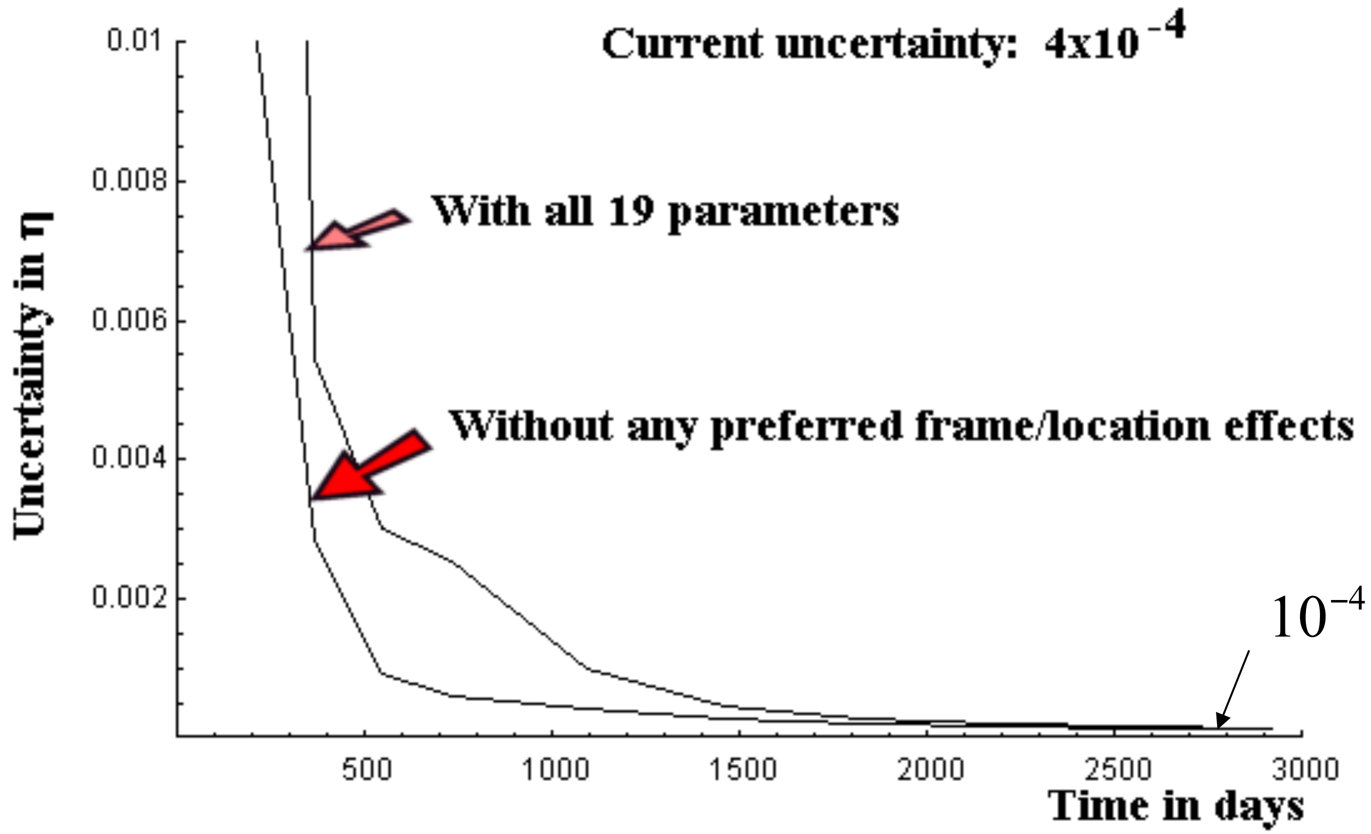
$$\delta r_r = 374.83\cos S_e - 4.87\cos(2S_e) - .35\cos(3S_e) - .04\cos(4S_e);$$
$$\delta r_t = -796.20\cos S_e + 6.94\cos(2S_e) + .43\cos(3S_e) + .04\cos(4S_e);$$

Mercury:

$$\delta r_r = 81.83\cos S_m - 0.09\cos(2S_m);$$
$$\delta r_t = -165.92\cos S_m + 0.12\cos(2S_m);$$

Effect of Saturn appears to be significant and is being studied.

Results for SEP-Violation Parameter η



Conclusions

Ranging to Mercury provides an opportunity for an independent check of numerous relativity parameters with comparable accuracy, as well as for determination of J_2 for the Sun with much improved accuracy. Orbital parameters should be determined with improved accuracy

Further work: Perturbations due to the asteroids, whose masses are sufficiently uncertain that they should be included as parameters to be determined.

Should the expression for the SEP-violation parameter be included?

$$\eta = 4\beta - 3\gamma - 1 \ldots$$

Can data be taken when the line of sight passes closer to the sun than 5^o?

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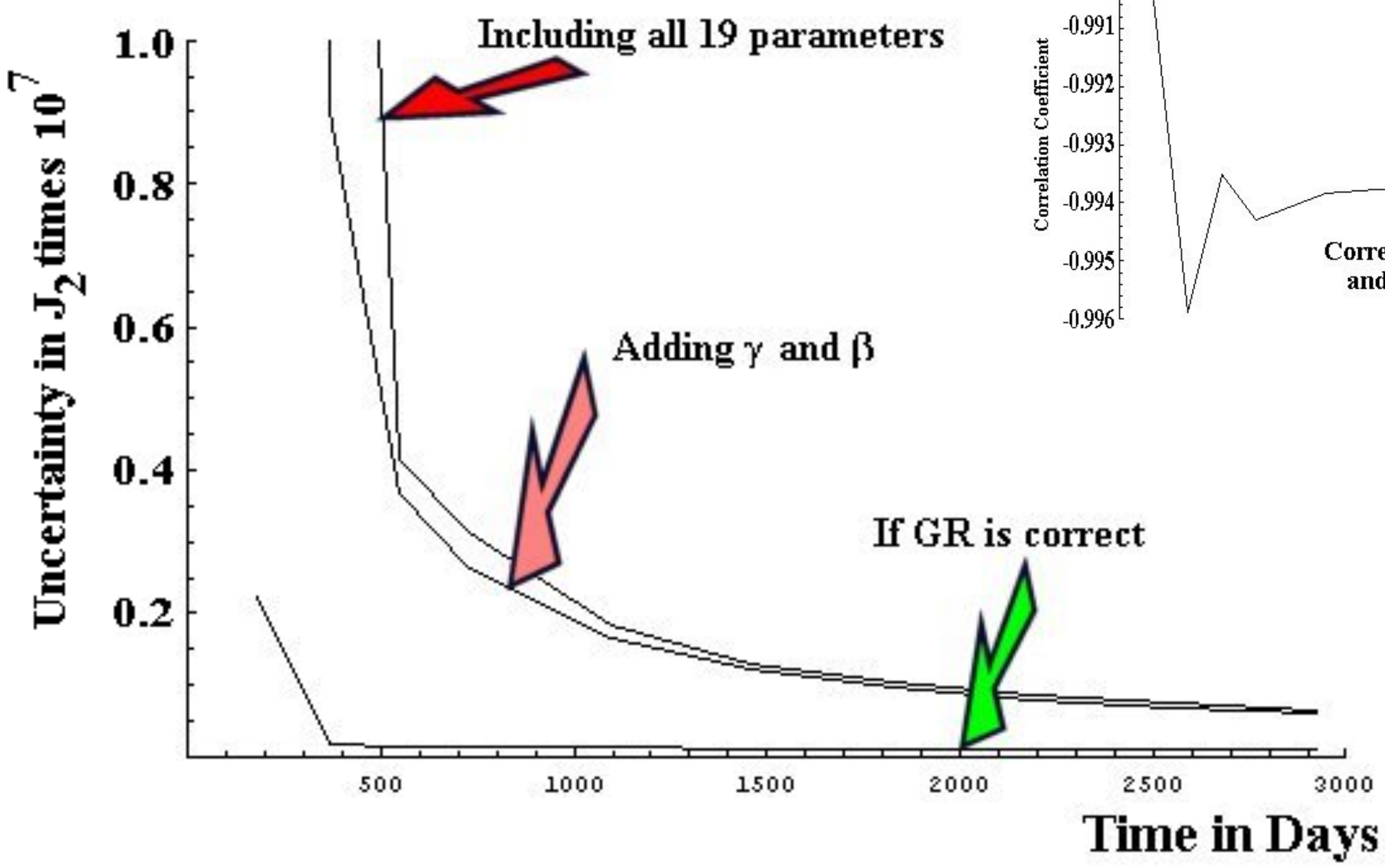
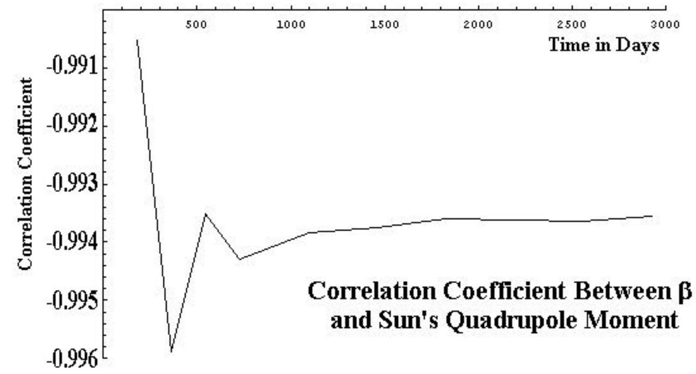
Calculation Results—Quadrupole Moment





Current Uncertainty: about 10^{-7}

Scientific Objectives of BepiColombo

— exploration of Mercury's unknown hemisphere;
— determination of Mercury's gravity field;
— investigation of the geological evolution of the planet;
— understanding the origin of Mercury's high density;
— analysis of the planet's internal structure; search for liquid outer core;
— investigation of the origin of Mercury's magnetic field;
— study of the planet's magnetic field interaction with the solar wind;
— characterization of the surface composition;
— identification of the composition of the radar bright spots in the polar regions;
— determination of the global surface temperature;
— determination of the composition of Mercury's vestigial atmosphere;
— determination of the source/sink processes of the exosphere;
— determination of the exosphere and magnetosphere structures;
— study of particle energization mechanisms in Mercury's environment;
— fundamental physics: verification of Einstein's theory of gravity.

--MORE Radio Science Instrument + about 14 other instruments
on Planetary Orbiter