LLR as Gravity Theory Laboratory

Gravity's Non-linearity (self coupling) --- the beta parameter
Frontier measurement today and can remain so.

Universality (composition independence) of Free-fall
Competitive with lab tests and can remain so.

dG/dt (and even superior spatial G gradient) constraint
Frontier measurement today and can remain so.

Any EP-violating cosmic accelerations?

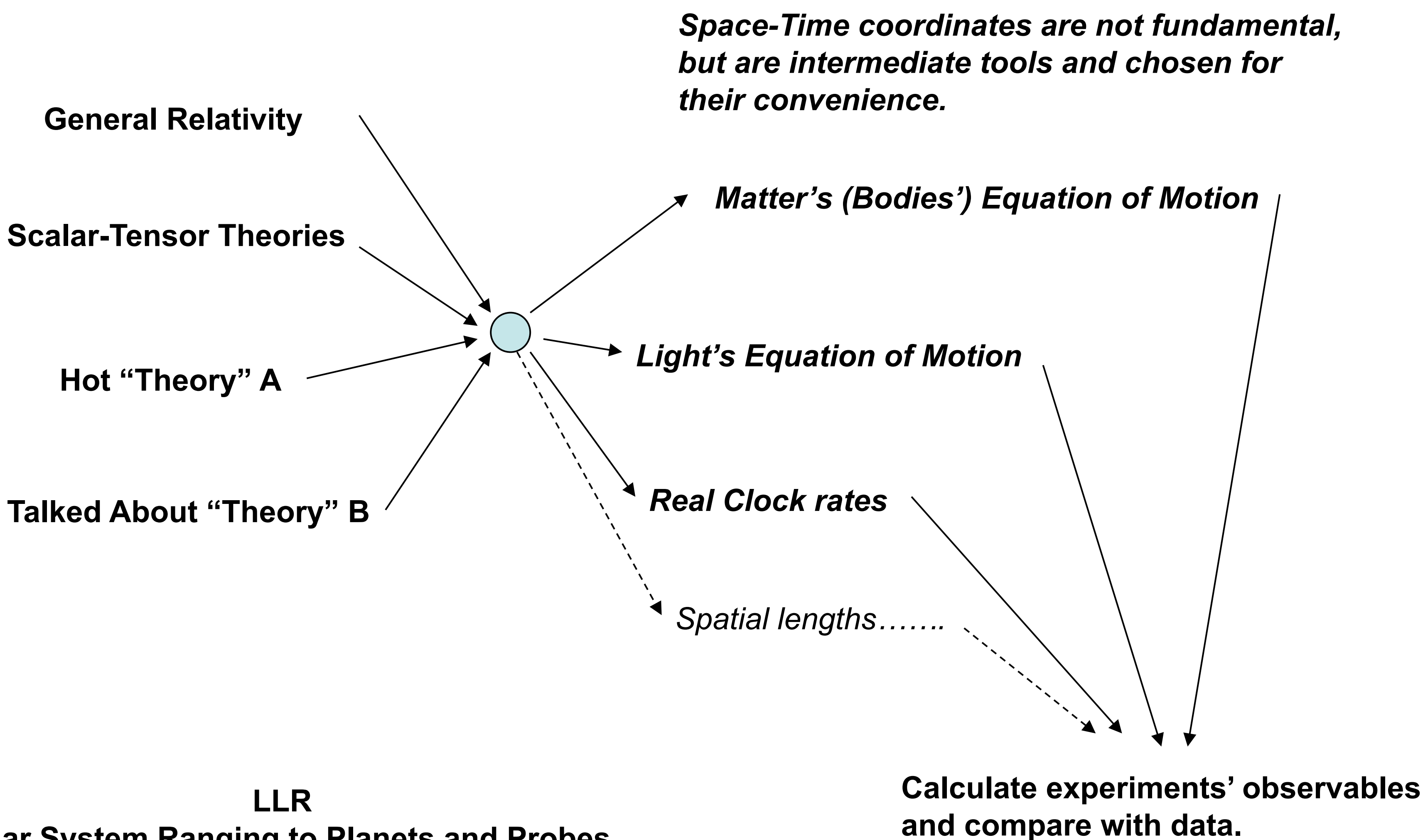
Measuring 2PN order gravity theory structure
M(I), M(G)… M(gamma), M(beta)……

LLR data's need for gravitomagnetic interaction.

Geodetic precession
Gravitational Spin-Orbit Force

Action-reaction equality in gravity

Lunar orbit's solar tidal energy and mass equivalence?



LLR
Solar System Ranging to Planets and Probes
Binary Pulsar timing

Cosmology
Galactic Structures and Dynamics
Neutron Star, Black Hole structures

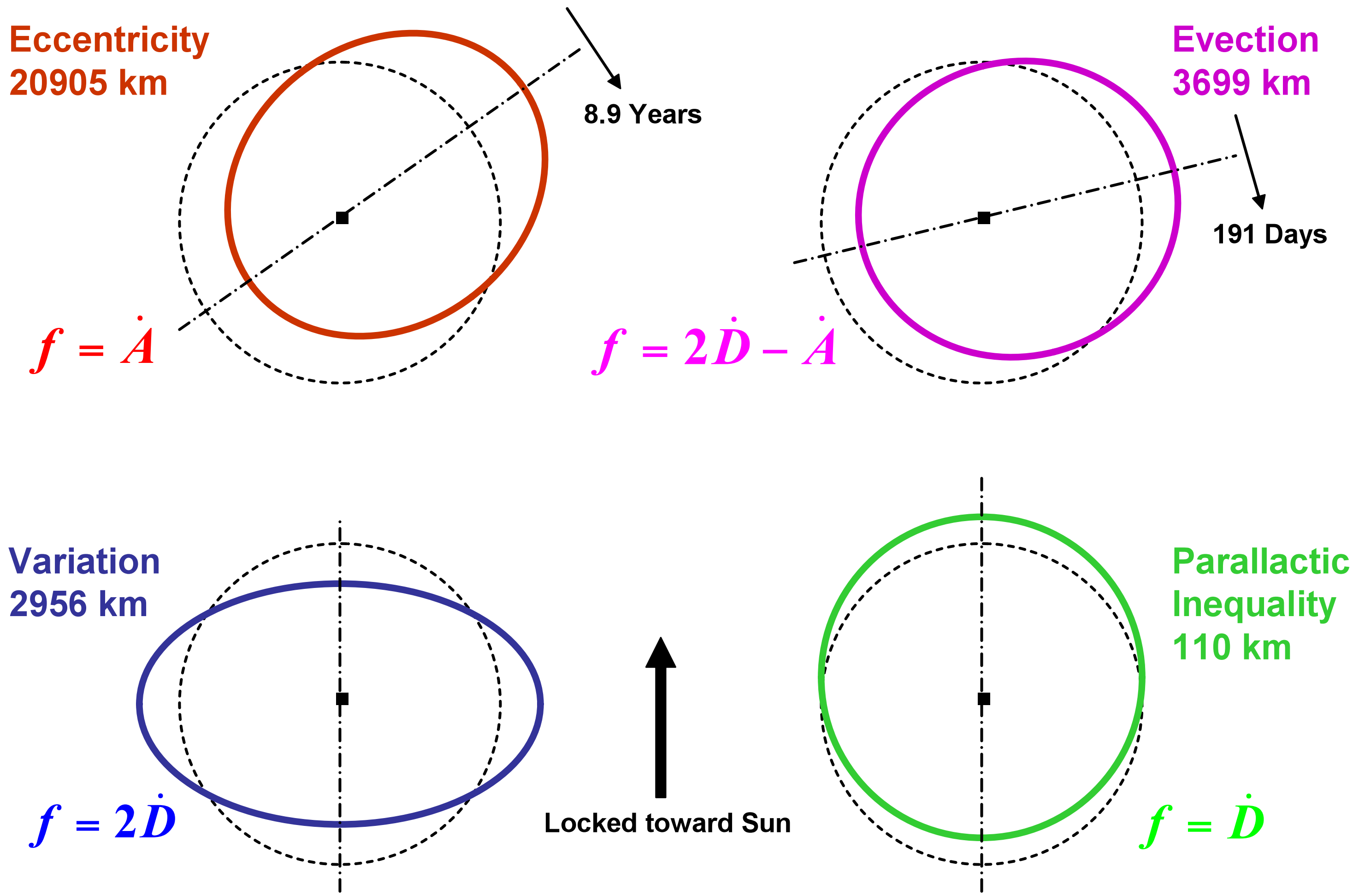
LLR data can be thought of as an empirical time series for the round trip times.

$$\boxed{\begin{aligned}\tau(t) &= \tau_0 + \dot{\tau}_0 (t - t_0) + \ldots.. \\ &+ \sum_n (A_n + \dot{A}_n (t - t_0) + \ldots.) \sin \int^t \omega_n(t)\, dt\end{aligned}}$$

$$\textit{with} \quad \omega_n(t) = \omega_n + \dot{\omega}_n (t - t_0) + \ldots.$$

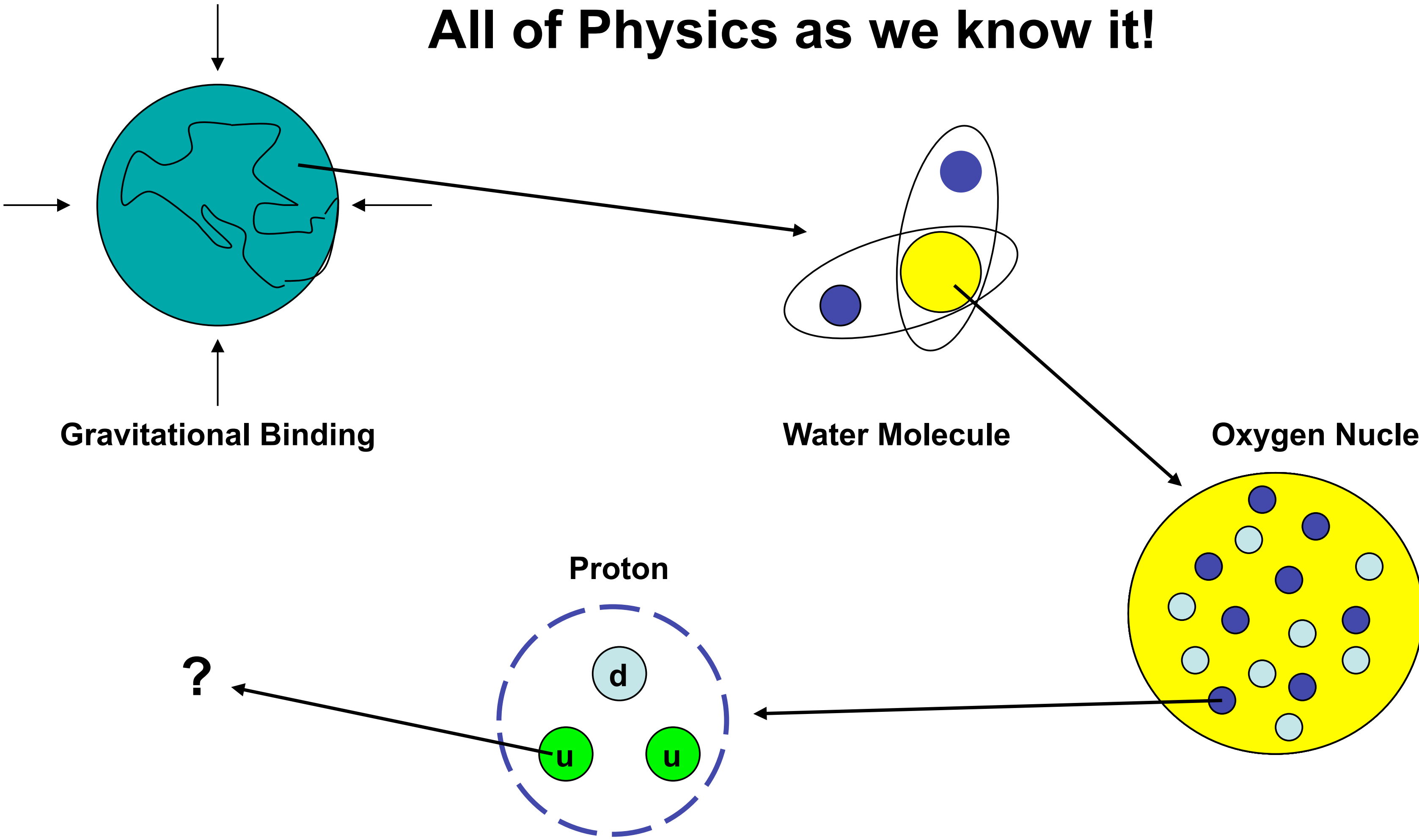
Amplitudes and frequencies and their time derivatives are then related to and fit by free initial conditions plus physical theory parameters including gravity theory.

Four Key Lunar Orbit Perturbations or "Inequalities"



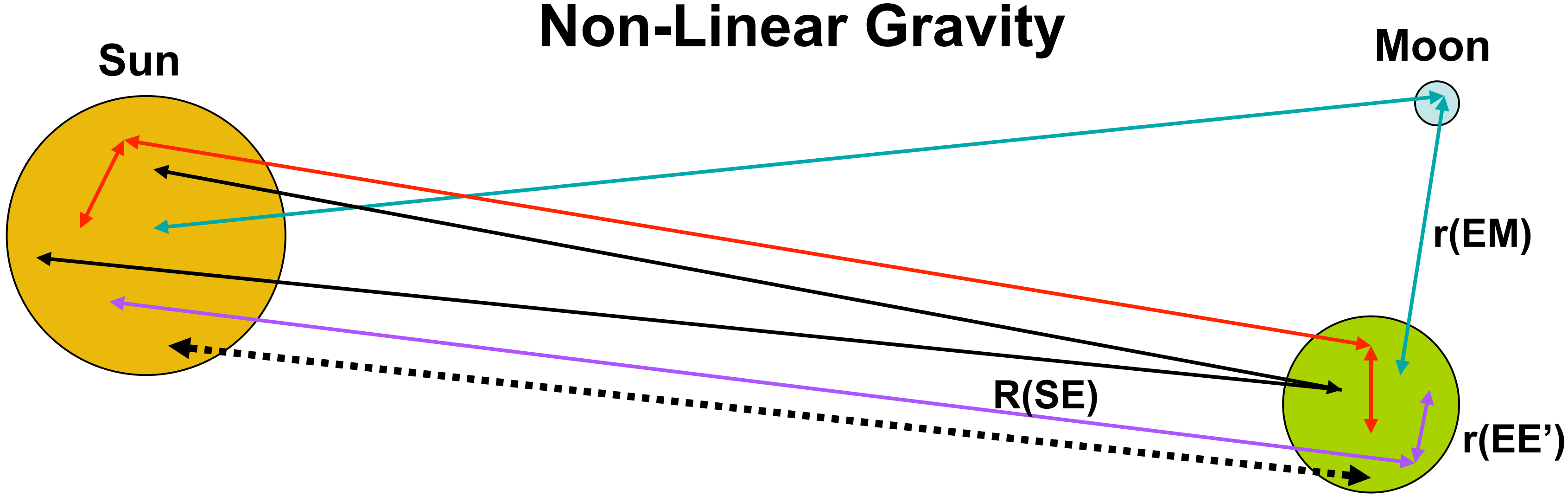
K. Nordtvedt, LLR, 25 July 2003

Earth's Mass-Energy consists of nuclear chromodynamic, electromagnetic, weak, kinetic, and gravity contributions. All of Physics as we know it!



How do all these forms of energy contribute to Earth's Inertial Mass and Gravitational Mass?

Non-Linear Gravity



$\frac{GM(S)m(E)}{R(SE)}$ is Newtonian interaction

$\frac{GM(S)m(E)}{R(SE)} \frac{GM(S)}{c^2 R(SE)}$ contributes to Einstein's classic planetary perihelion precession (in black).

$\frac{Gm(E)m(M)}{r(EM)} \frac{GM(S)}{c^2 R(SE)}$ is 3-body interaction contributing to lunar orbit EP test (in blue).

$\frac{Gm(E)m(E')}{r(EE')} \frac{GM(S)}{c^2 R(SE)}$ is 3-body interaction proportional to Earth's gravitational binding energy (in violet).

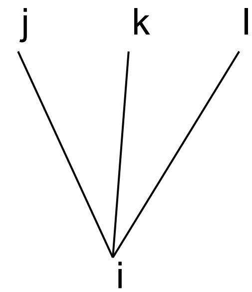
This interaction is 50 times larger than the lunar orbit interaction, resulting in:

$$\frac{M(G)}{M(I)} = \frac{M(NG) + (4\beta - 2 - \gamma)\, U(G)/c^2}{M(NG) + U(G)/c^2}$$

2nd post-Newtonian order 4-body interaction (in red) --- is a $5\ 10^{-10} \times 4\ 10^{-6} = 2\ 10^{-15}$ EP test

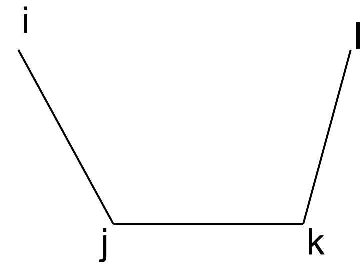
2^{nd} Post-Newtonian order Gravity Interactions

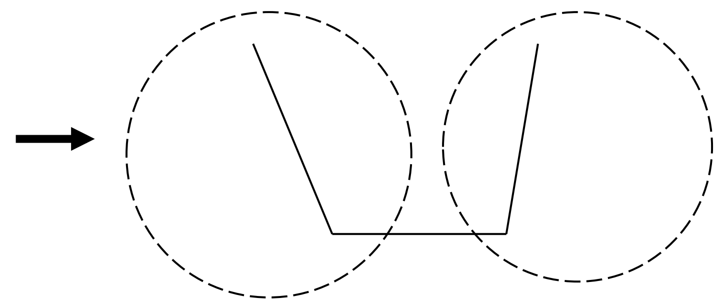
$$\frac{G^3}{c^4}\sum_{ijkl}\frac{m_i\ m_j\ m_k\ m_l}{r_{ij}\ r_{ik}\ r_{il}}$$



Compactifying pairs of mass sources into two bodies as indicated below leads to:

$$\frac{G^3}{c^4}\sum_{ijkl}\frac{m_i\ m_j\ m_k\ m_l}{r_{ij}\ r_{jk}\ r_{kl}}$$





Gravitational Self Energy pulling on Gravitational Self Energy

For the Sun-Earth interaction this 2nd Post-Newtonian order interaction leads to potential EP violation in LLR at the level:

$$4\,10^{-6}\ x\ 5\,10^{-10} = 2\,10^{-15}$$

Consequences of Gravity's Non-Linearity

$$\frac{G(\vec{R}) - G_\infty}{G_\infty} = \frac{G\left(M(G) + \gamma M(\gamma) - (4\beta - 2)M(\beta)\right)}{c^2 R}$$

$$\delta \vec{a} = -c^2 \frac{1}{M(I)} \frac{\partial M(I)}{\partial G} \vec{\nabla} G$$

$$\frac{M(G)}{M(I)} = 1 + \frac{(4\beta - 2)M(\beta) - \gamma M(\gamma) - M(G)}{M(G)} \frac{U(G)}{M(I)c^2}$$

$$\frac{M(G)}{M(I)} = 1 + \frac{(4\beta - 2)M(\beta) - (1 + \gamma)M(I)}{M(G)} \frac{U(G)}{M(I)c^2} \quad \text{with extended Lorentz Invariance}$$

$$\frac{M(G)}{M(I)} = 1 + (4\beta - 3 - \gamma) \frac{U(G)}{M(I)c^2} \left(+ \kappa \frac{T(rot)}{M(I)c^2} \left[I - 3\hat{p}\hat{p}\right]_{ab} \right) \quad \text{in lowest order}$$

For Earth, $U(G)/Mc^2 \simeq 5\,10^{-10}$ **and** $T(rot)/Mc^2 \simeq 4\,10^{-13}$

γ, β, $M(\gamma)$, and $M(\beta)$ are all subject to 2nd and higher order structure of theory

Post-Newtonian Gravity for Gravitationally-Bound Bodies

$$L = -\sum_i M(I)_i \left[1 - \frac{1}{2}v_i^2 - \frac{1}{8}v_i^4\right] + \frac{1}{2}\sum_{ij}\frac{\Gamma_{ij}}{r_{ij}}\left[1 - \frac{1}{2}\vec{v}_i \cdot \vec{v}_j - \frac{1}{2}\vec{v}_i \cdot \hat{r}_{ij}\hat{r}_{ij} \cdot \vec{v}_j\right]$$

$$\frac{1}{4}\sum_{ij}\frac{2\gamma\ \Theta_{ij} + \Gamma_{ij}}{r_{ij}}\left(\vec{v}_i - \vec{v}_j\right)^2 + \left(\frac{1-2\beta}{2}\right)\sum_{ijk}\frac{\Gamma_{ijk}}{r_{ij}r_{ik}} + \ldots.$$

Limits of couplings $\Gamma_{ij}, \Gamma_{ijk}, \Theta_{ij}$ as bodies become test bodies:

$$\Gamma_{\mathrm{ij}} \to G\ M(G)_i\ m_j \to G\ m_i\ m_j$$

$$\Theta_{ij} \to G\ M(\gamma)_i\ m_j \to G\ m_i\ m_j$$

$$\Gamma_{ijk} \to G^2\ \Lambda_{jk}\, m_i \to G^2\ M(\beta)_k\, m_j\, m_i \to G^2\ m_i\, m_j\, m_k$$

$$\Gamma_{ijk} \to G^2\ \Lambda'_{ij}\, m_k \to G^2\ M(\beta')_i\, m_j\, m_k$$

$$M(I)_i \to m_i$$

"weak field" versus "strong field" --- a bit of Red Herring?

Every non-linear order of the theory is "in" the G, γ, β...... PPN coefficients

$U(\vec{r}_{SS}, t) =$ *solar system potential* $\qquad U =$ *surrounding matter potential*

$$g_{00} = 1 - 2(U + U_{SS}) + 2\beta(U + U_{SS})^2 + \sum_{n=3} 2c_n(U + U_{SS})^n + \textit{more...}$$

$$= [1 - 2U + 2\beta U^2 + \sum_{n=3} 2c_n U^n] \qquad \textit{Time renormalization, } t \Rightarrow \tau(\vec{R}, T)$$

$$-\, 2U(\vec{r}_{SS}, t)\,[1 + 2\beta U + \sum_{n=3} n c_n U^{n-1}] \qquad G \textit{ renormalization, } G \Rightarrow G(\vec{R}, T)$$

$$+\, U(\vec{r}_{SS}, t)^2\,[2\beta + \frac{1}{2}\sum_{n=3} n(n-1) c_n U^{n-2}] \qquad \beta \textit{ renormalization, } \beta \Rightarrow \beta(\vec{R}, T)$$

But from the spatial metric non-linearity the proper length variable gets rescaled:

$$\vec{r}_{SS} = \vec{\rho}_{SS}\,[1 - \gamma U + \sum_{n=2} d_n U^2]$$

The metric field in the solar system is not perturbative from an empty universe;
it is perturbative from it's place in the surrounding actual universe.
The lowest order PPN coefficients obtain contributions from every non-linear order

These indicated PPN coefficients will in general be renormalized by matter surrounding Solar System … just as G is renormalized.

$$g_{oo} = 1 - 2U + U^2 + (2\beta - 1)\frac{G^2}{c^4}\sum_{ij}\left[\frac{\Lambda_{ij}}{r_i\ r_j} + 2\frac{\Lambda'_{ij}}{r_i\ r_{ij}}\right] + \ldots.$$

$$g_{ab} = -\left[1 + 2\gamma\frac{G}{c^2}\sum_i \frac{M(\gamma)_i}{r_i}\right]\delta_{ab} + \ldots.$$

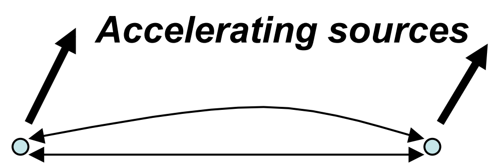
$$with\ \ U = \frac{G}{c^2}\sum_i \frac{M(G)_i}{r_i} \quad and \quad r_i = |\vec{r} - \vec{r}_i|$$

These indicated mass and mass-squared parameters will in general individually differ from body inertial mass energies by bodies' gravitational binding energies.

How does an Object's internal interaction energy get converted into Inertial Mass?

From Landau and Lifshitz, the $1/c^2$ Lagrangian for a collection of charged particles is:

$$L = -\sum_i m_i \left(c^2 - \frac{v_i^2}{2} - \frac{v_i^4}{8c^2} \right) - \frac{1}{2} \sum_{ij} \frac{e_i e_j}{r_{ij}} \left(1 - \frac{\vec{v}_i \cdot \vec{v}_j}{2c^2} - \frac{\vec{v}_i \cdot \hat{r}_{ij}\ \hat{r}_{ij} \cdot \vec{v}_j}{2c^2} \right) + ...$$



Using equations of motion from least action $\dfrac{d}{dt}\dfrac{\partial L}{\partial \vec{v}_i} = \dfrac{\partial L}{\partial \vec{r}_i}$

Inertial mass of system of interacting charges accelerating as a whole is:

$$M = \sum_i m_i \left(1 + \frac{v_i^2}{2c^2} \right) + \frac{1}{2c^2} \sum_{ij} \frac{e_i e_j}{r_{ij}} + \frac{1}{c^2} \left[\sum_i m_i\ \vec{v}_i\ \vec{v}_i + \frac{1}{2} \sum_{ij} \frac{e_i e_j}{r_{ij}^3}\ \vec{r}_{ij}\ \vec{r}_{ij} \right]$$

The inertia of the electric interaction energy results from the field lines of mutually accelerated charges developing curvature which then yields net forces proportional to that acceleration-driven field line curvature .

Same thing happens with nuclear forces, QCD forces, gravity forces.....

$$\frac{G(r)-G(\infty)}{G(\infty)} = (4\beta - 3 - \gamma)\frac{U(r)}{c^2}$$ **valid in perturbative metric gravity**

$$\frac{\vec{\nabla} G}{G} = (4\beta - 3 - \gamma)\frac{\vec{\nabla} U}{c^2}$$ **is another interpretation of** $M(G)/M(I) \neq 1$

$$\left|\frac{\vec{\nabla} G}{G}\right| \leq 2\,10^{-4}\ \vec{g}(Sun)/c^2$$ **from LLR**

$$\boxed{\frac{dG/dt}{G} = (4\beta - 3 - \gamma)\frac{dU/dt}{c^2} \quad \Rightarrow \quad (4\beta - 3 - \gamma)\, H}$$

LLR constrains dG/dt **via frequency shifts in lunar motion:**

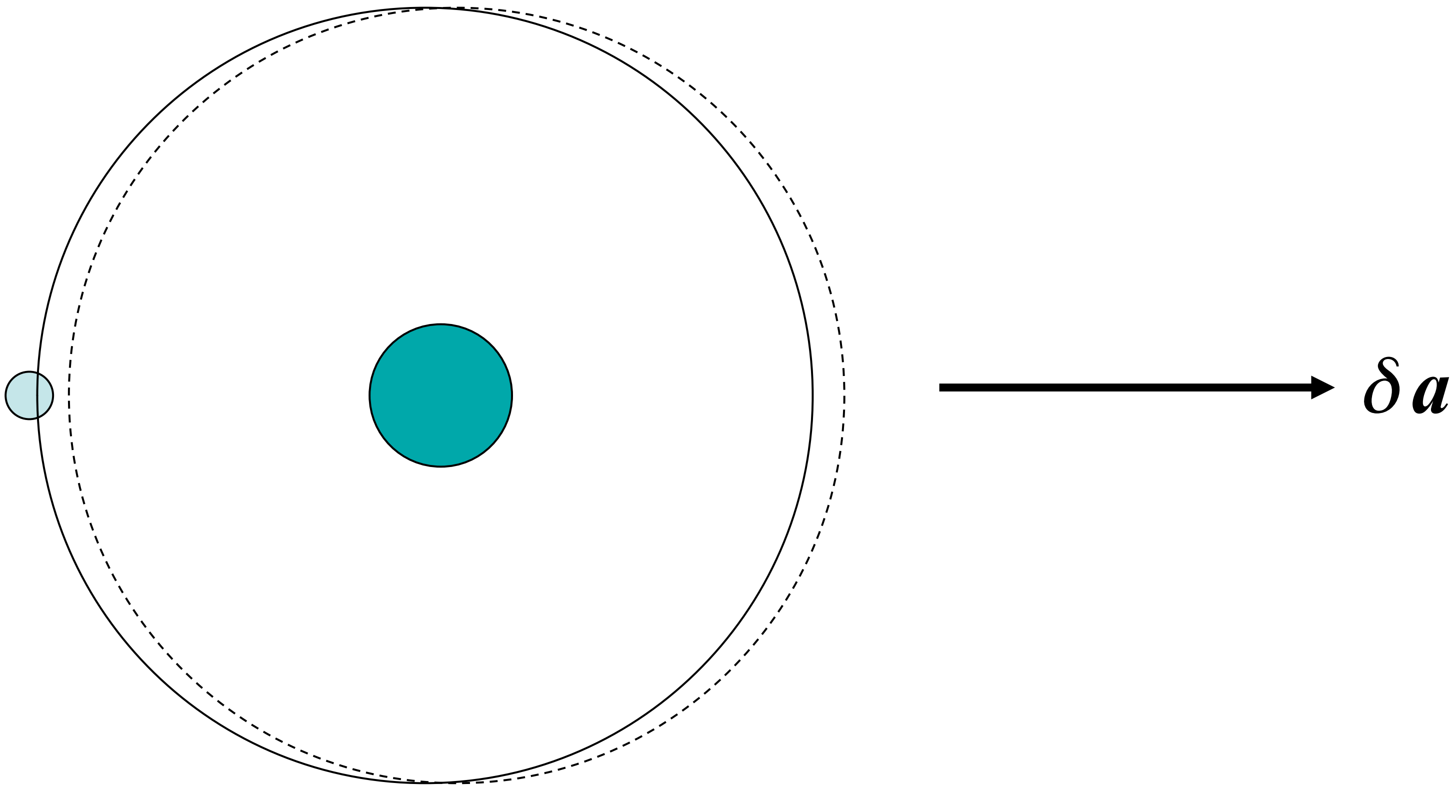
$$f(t) = f(t_o)\,\{1 + 2dG/Gdt(t - t_o)\}$$

$$\theta(t) = f(t_o)\,(t - t_o) + f(t_o)\, dG/Gdt\,(t - t_o)^2$$

$$r(t) = r(t_o)\,\{1 - dG/Gdt\,(t - t_o)/2\}$$

$$dG/H dt \leq 10^{-3}$$ **from LLR**

Cosmic EP-violating acceleration field?
Source? Dark Matter? Dark Energy? 72 – 24 – 4 !



$$\delta r = \frac{3}{2}\frac{\delta a}{\dot{L}(\dot{L}-\dot{A})} \qquad \dot{L}-\dot{A} = \frac{2\pi}{8.9\,years}$$

$$|\,\delta a\,| \le 10^{-14}\ cm/s^2 \quad \simeq \quad 2\,10^{-7} \quad cH$$

Cosmic (Sidereal) EP-violating Acceleration?

$$x = \frac{3}{2}\frac{\delta a}{\omega(\omega-\omega_o)}\cos\omega(t-t_c) \qquad \omega-\omega_o = 2\pi/(8.9\,years)$$

ω is sidereal lunar frequency, ω_o is anomalistic frequency

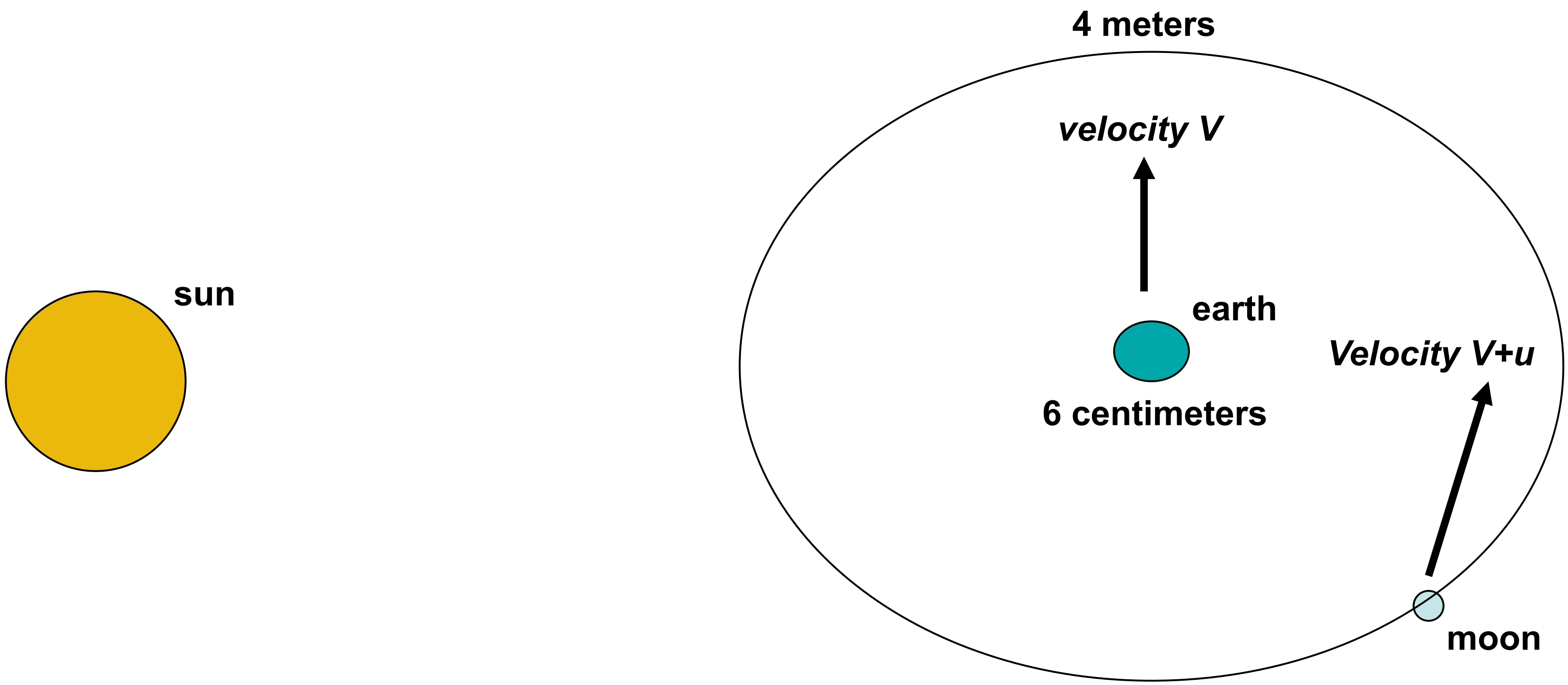
For $x = 1\ mm$ a $\delta a = 4\,10^{-15}\ cm/s^2$ is detectable

But for $U(\alpha)/Mc^2 = 10^{-3}$ difference between Earth and Moon

this allows detection of cosmic gradient of $\alpha = e^2/\hbar c$ at level

$$\frac{\nabla\alpha}{\alpha} = 4\,10^{-33}\ cm^{-1} = 6\,10^{-5}\ 1/(cT_U)$$

This tests whether dark matter to ordinary matter interaction fulfills the EP? Recall ripples in cosmos are at the 10^{-5} level.



Why is Earth contracted 6 cm. in diameter parallel to its motion? Because the electrodynamics, including magnetic forces, Lorentz contracts the atoms and molecules…..

Why is Moon's orbit contracted 4 meters in diameter parallel to its motion? Because the gravitational dynamics , including gravitomagnetic forces, Lorentz contracts the orbit dynamics.

$$\vec{a}_M = \vec{g}_E \left(1 + Order\left[u_E^2 \;\; u_E u_M \;\; u_M^2 \right] / c^2 + ...\right) \;\; + \;\; \vec{g}_S \left(1 + Order\left[u_M^2 \right] / c^2 + ...\right)$$

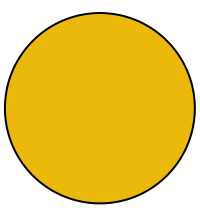
Orbit Lorentz contraction, perigee, perihelion, geodetic precession+SO, M(G)/M(I)

Gravitational Spin-Orbit Force

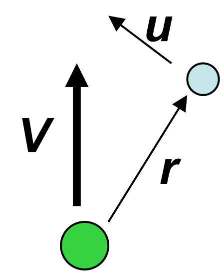
A spinning body does not move on same gravitational trajectory as a test body.

A body’s angular momentum couples to the Newtonian gravitational gradient.

The orbitting Moon can be viewed as a spinning body.



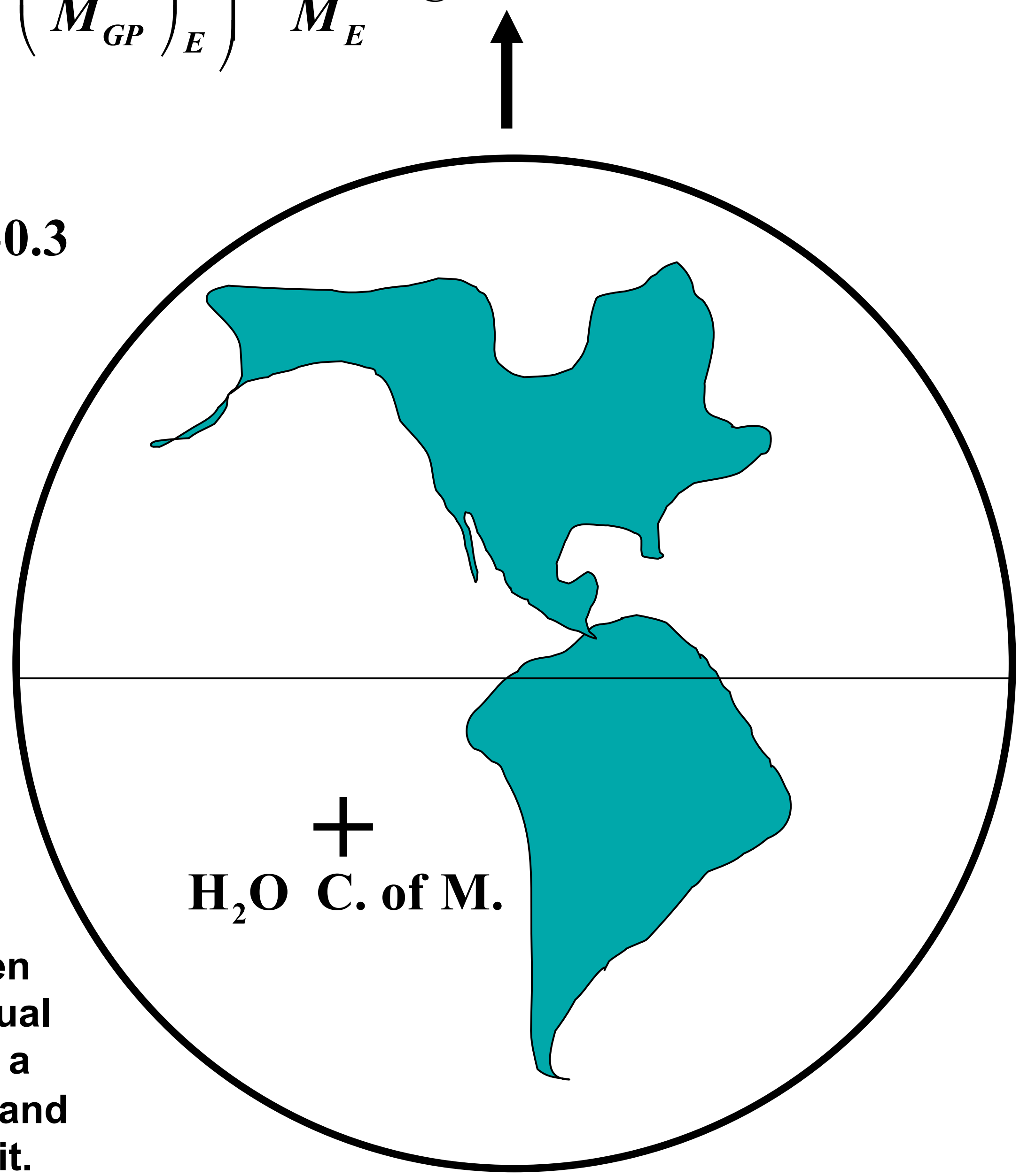
$$a = \frac{GM}{c^2 R^3}(ru)V \approx 8\,10^{-13}\ g(Sun)$$



$$a = C_{ISO}\ \sin Lat \left(\left(\frac{M_{GA}}{M_{GP}} \right)_{H_2O} - \left(\frac{M_{GA}}{M_{GP}} \right)_{E} \right) \frac{M_{H_2O}}{M_E}\ g$$

$$\frac{M_{H_2O}}{M_E} \simeq 2.3\ 10^{-4} \qquad \sin Lat \simeq -0.3$$

$$\left| \left(\frac{M_{GA}}{M_{GP}} \right)_{H_2O} - \left(\frac{M_{GA}}{M_{GP}} \right)_{E} \right| \leq 3\ 10^{-13}$$



If the gravitational forces between Earth's water and core are not equal and opposite, Earth experiences a self-acceleration along polar axis, and consequently perturbs lunar orbit.

Does Earth's $M(G)/M(I)$ ratio become anomalous at the level of lunar orbit's Solar Tidal Energy?

Strength of Newtonian Solar Tidal Energy $\frac{1}{2}\vec{r}\cdot Q\cdot\vec{r}/c^2 \simeq \boxed{7\,10^{-14}}$

$$Q = \frac{GM_S}{R^3}\left[I - 3\,\hat{R}\,\hat{R}\right] + ...$$

Lunar orbit's Newtonian level energy function $e = \frac{1}{2}v^2 - u(r) + \frac{1}{2}\vec{r}\cdot Q\cdot\vec{r}$

is a constant of the motion in the limit the rotation of the Sun's Newtonian tidal tensor Q can be neglected.

$u(r) \simeq Gm/r$ is the earth's Newtonian potential.

But a solar tidal energy contribution to orbit energy e, in addition to constant part, oscillates at twice the synodic frequency, so therefore the "local" energy $\frac{1}{2}v^2 - u(r)$ does as well.

Lunar orbit's solar tidal energy does not "fall" with the Moon as do the "local" contributions to lunar orbit energy. So exactly how this tidal energy affects moon's effective $M(G)/M(I)$ ratio requires separate investigation in Post-Newtonian Metric Gravity.

LLR is close approaching the level of precision needed to measure this solar tidal contribution.

$1/c^2$ Electrodynamics versus PPN Metric Gravity

$$L = -\sum_i m_i \left(c^2 - \frac{v_i^2}{2} - \frac{v_i^4}{8c^2} \right) - \frac{1}{2}\sum_{ij} \frac{e_i e_j}{r_{ij}} \left(1 - \frac{\vec{v}_i \cdot \vec{v}_j}{2c^2} - \frac{\vec{v}_i \cdot \hat{r}_{ij}\ \hat{r}_{ij} \cdot \vec{v}_j}{2c^2} \right)$$

versus

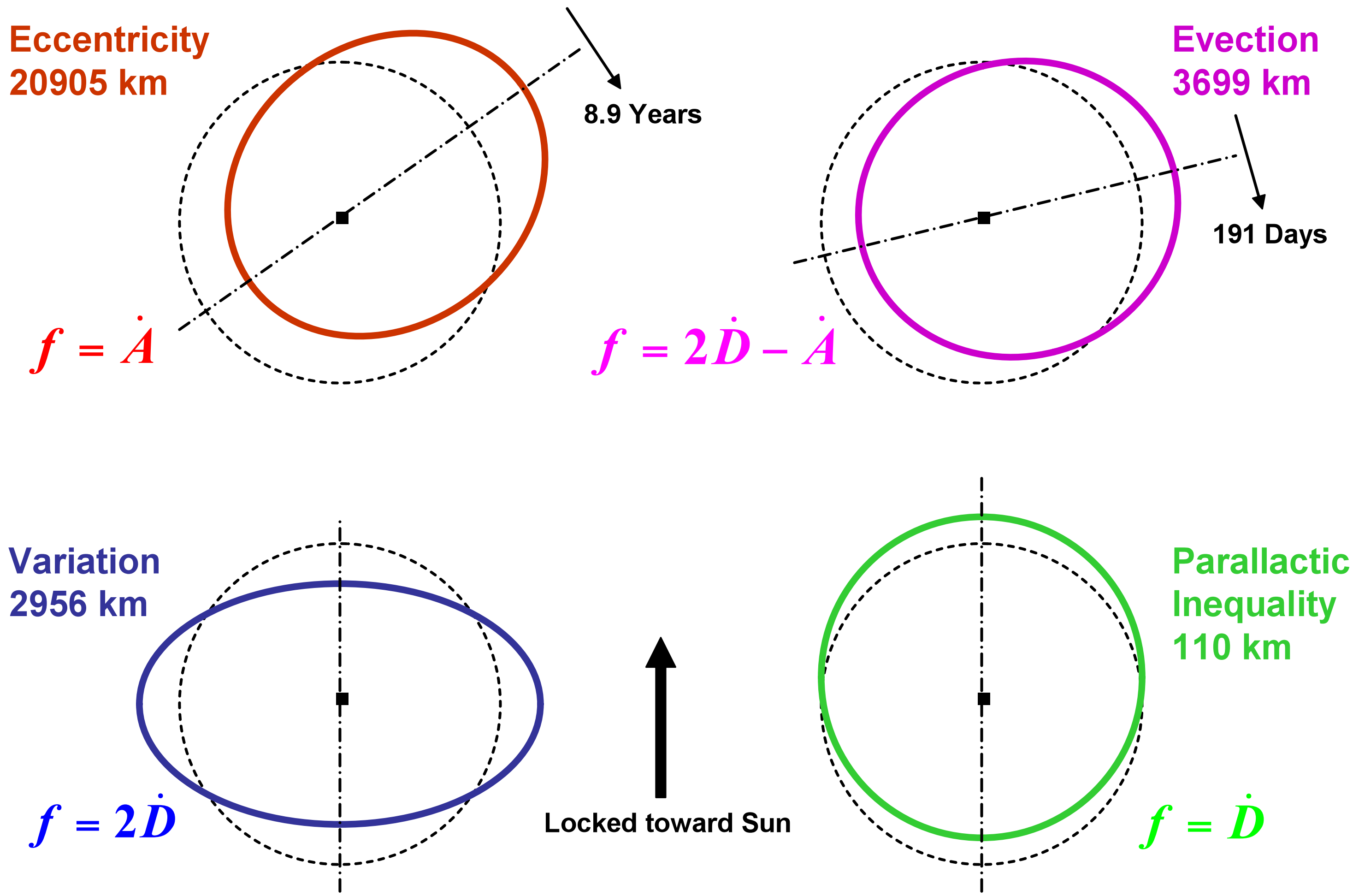
$$L = -\sum_i m_i \left(c^2 - \frac{v_i^2}{2} - \frac{v_i^4}{8c^2} \right) + \frac{1}{2}\sum_{ij} \frac{G m_i m_j}{r_{ij}} \left(1 - \frac{\vec{v}_i \cdot \vec{v}_j}{2c^2} - \frac{\vec{v}_i \cdot \hat{r}_{ij}\ \hat{r}_{ij} \cdot \vec{v}_j}{2c^2} \right)$$

$$+ \left(1 + 2\gamma\right) \frac{1}{4c^2} \sum \frac{G m_i m_j}{r_{ij}} \left(\vec{v}_i - \vec{v}_j \right)^2 + \left(1 - 2\beta\right) \frac{1}{2c^2} \sum_{ijk} \frac{G^2 m_i m_j m_k}{r_{ij}\ r_{ik}}$$

$$M(I) = \sum_i m_i \left(1 + \frac{1}{2c^2} v_i^2 \right) - \frac{1}{2c^2} \sum_{ij} \frac{G m_i m_j}{r_{ij}} - \frac{1}{2c^2} \left(1 + 2\gamma\right) \sum_i m_i\ \vec{r}_i \cdot Q \cdot \vec{r}_i$$

$$+ \frac{1}{c^2} \left[\sum_i m_i\ \vec{v}_i\ \vec{v}_i - \frac{1}{2} \sum_{ij} \frac{G m_i m_j}{r_{ij}}\ \hat{r}_{ij}\ \hat{r}_{ij} \right]$$

Four Key Lunar Orbit Perturbations or “Inequalities”



$$\vec{a}_M = \vec{g}_E - [Q_S]\cdot\vec{r}_M \qquad [Q_S] = \frac{GM}{R^3}\left[I - 3\hat{R}\hat{R}\right]$$

yields Newton's Variational Orbit with cos2D perturbation

In limit Sun's Tidal Tensor $[Q_S]$ is not rotating

there a lunar orbit energy constant of the motion:

$$e = \frac{1}{2}v_M^2 + U_E(r_M) + \frac{1}{2}\vec{r}_M\cdot[Q_S]\cdot\vec{r}_M \qquad Q\, r_M^2 / c^2 \approx 7\; 10^{-14}$$

Does $M(I) = m\left(1 + e/c^2\right)$? How about $M(G)$?

– – – – – – – – – – – – – – – – – – – –

$$E\;\vec{R}_{CoE} = \sum_i m_i \left(1 + \frac{v_i^2}{2c^2}\right)\vec{r}_i - \frac{G}{4c^2}\sum_{ij}\frac{m_i\; m_j}{r_{ij}}\left(\vec{r}_i + \vec{r}_j\right)$$

with $$E = \sum_i m_i\left(1 + \frac{v_i^2}{2c^2}\right) - \frac{G}{2c^2}\sum_{ij}\frac{m_i\; m_j}{r_{ij}} \qquad d\vec{R}_{CoE}/dt = 0$$

Sun's metric field expansion contains gravitational mass $M(G)$ but also additional mass parameters entering various metric potentials

$M(\gamma)$, $M(\beta)$, *etc*.

$$[M(G)/M(I)]_{earth} = 1 + (4\beta - 3 - \gamma)\, U/Mc^2 \equiv 1 + \eta\, [U/Mc^2]_{earth}$$

U being a body's internal gravitational binding energy

$$\textit{But} \quad \beta - 1/2 \rightarrow (\beta - 1/2)\,[M(\beta)/M(G)]_{sun} \quad \textit{and} \quad \gamma \rightarrow \gamma\,[M(\gamma)/M(G)]_{sun}$$

$$\textit{with } M(\gamma)/M(G) = 1 + \sigma\, U/Mc^2 \quad \textit{and} \quad M(\beta)/M(G) = 1 + \sigma'\, U/Mc^2$$

σ and σ' are combinations of 2nd PPN order theory parameters

Sun's gravitational binding is $[U/Mc^2]_{sun} \cong 4\,10^{-6}$

So experiments which reach precision level $4\,10^{-6}$ in measuring γ and β begin measuring 2nd PPN order σ and σ'

Can LLR reach this level?

What viable theory types can deviate at 2nd PPN but not 1st PPN Order?

Space and Time Variation of Fundamental 'Constants'

$$\vec{f} = U(\alpha)\,\vec{\nabla}\alpha/\alpha \qquad U(\alpha)\ \textit{being electric energy content of body's matter}$$

$$\textit{Then}\quad \vec{a} = [U(\alpha)/Mc^2]\quad c^2\vec{\nabla}\alpha/\alpha$$

$$U(\alpha)/M\ \textit{scales as}\ Z^2/A^{4/3}\ \textit{among the element nuclei}$$

$$\textit{From LLR, and the}\ [U(\alpha)/Mc^2]\ \textit{difference of}\ 10^{-3}\ \textit{between Earth and Moon}$$

$$\vec{\nabla}\alpha/\alpha \le 1.6\,10^{-10}\ \vec{g}(sun)/c^2$$

Spectral lines from very distant objects in the universe suggest a bound on

time dependence of α $\qquad \dfrac{1}{\alpha}\dfrac{d\alpha}{dt} \le 10^{-5}\ \textit{Hubble Rate}$

Strongest space-time constraint on fundamental constant variation is from LLR?

(although $\vec{\nabla}$ *versus* d/dt comparison will be theory-dependent)

Newtonian Virials

$$\int_0^T dt \sum_i m_i\, \vec{a}_i\, \vec{r}_i = \int_0^T dt \sum_{ij} \vec{f}_{ij}\, \vec{r}_i + \int_0^T dt\, \vec{f}(ex)_i\, \vec{r}_i$$

Integrate left hand side by parts for time interval T

$$\int_0^T dt \left[\sum_i m_i\, \vec{v}_i\, \vec{v}_i + \sum_{ij} \vec{f}_{ij}\, \vec{r}_i + \sum_i \vec{f}(ex)_i\, \vec{r}_i \right] = \left[\sum_i m_i\, \vec{v}_i\, \vec{r}_i \right]_0^T$$

If right hand side does not grow linear in T --- equilibrium --- then left hand side integrand is on average zero. That is the spatial tensor virial with its auxiliary scalar virial. For $\vec{f}_{ij} = -\vec{f}_{ji}$

$$\left\langle \sum_i m_i\, \vec{v}_i\, \vec{v}_i + \frac{1}{2} \sum_{ij} \vec{f}_{ij}\, \vec{r}_{ij} \right\rangle \cong -\left\langle \sum_i \vec{f}(ex)_i\, \vec{r}_i \right\rangle$$

For isolated body right hand side vanishes

Virial Theorem and Lunar Orbit's Eccentric Energy

The eccentric and evective motion of the Moon's orbital motion produces oscillations of the Virial = $2T + V$ about its average value of zero. The vanishing of the scalar and tensor Virials are required for inertial and gravitational mass equivilents of this orbital energy being equal.

Eccentric energy is of order em/r with evective energy somewhat smaller. Because eccentric energy oscillates at sidereal frequency, possible range perturbations result at annual frequency and twice sidereal minus annual frequency. Characteristic size is $e\ g_S r^2 / c^2 = .5\ mm$. Smaller evective motion contributions are at slightly different frequencies.

These "perturbations" are probably coordinate effects; the proper distance coordinate $\vec{\rho}$ in the variable Solar Potential differs from the isotropic $\vec{r}$ coordinate as

$$\vec{\rho} = \vec{r}\ \left(1 + \gamma\ \vec{g} \cdot \vec{r} / c^2\right) - \gamma\ \vec{g}\ r^2 / 2c^2$$