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Feasibility of observations below 40 MHz with a *DARE*-like system

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Feasibility of observations below 40 MHz with a *DARE*-like system

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1 Introduction

The current DARE band is 40–120 MHz, a range which was picked with the intention of maximizing the chance of recovering turning points B, C and D of the 21-cm signal (see Fig. 1), and constrained by the need to produce an antenna with a smooth frequency response that could fit in a rocket fairing, wouldn't need deploying, etc. At this stage it may be worth revisiting the frequency range. Firstly,

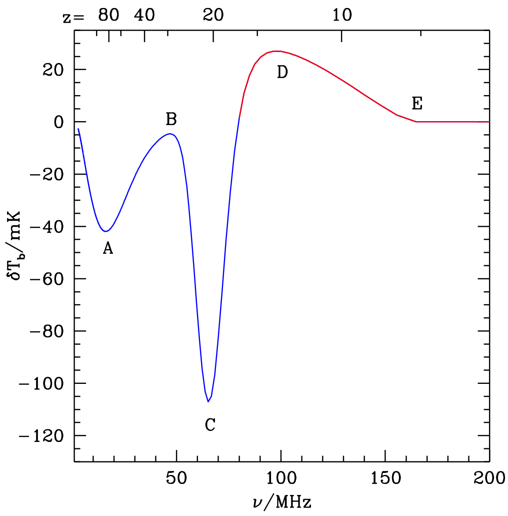


Figure 1: Evolution of the 21-cm brightness temperature in our fiducial model (Pritchard and Loeb, 2010), with the turning points labelled.

detecting turning point B is difficult for *DARE*, requiring $\gtrsim 2000$ h to bring the lower end of the 95% confidence interval on the inferred frequency above 40 MHz for the fiducial signal model. Is it worth risking missing turning point D in order to increase the chance of getting turning point B, given that there is arguably more chance of ground-based observations reaching D anyway? This would suggest reducing the frequency range to something like 35–105 MHz, assuming that a factor of three in frequency is roughly what's achievable. Secondly, if a radical redesign of the antenna could produce a much larger frequency range, say 20–120 MHz, possibly at the expense of introducing frequency structure into the response, could this allow constraints on the true 'Dark Ages', from a pure sensitivity point of view?

2 Pure noise calculation

We start by computing the integration time required to reach a reasonable thermal noise level. The noise depends on the intensity of the foregrounds, which increases at low frequency. We start by assuming a simplified version of the instrument response, ignoring correlations of the noise, attenuation, etc.:

$$T_{\rm ant} = (1 - |\Gamma|^2)T_{\rm sky} + (1 + |\Gamma|^2)T_{\rm rcv} \; . \qquad (1)$$

T_{ant}, T_{sky} and T_{rcv} are the antenna, sky and receiver temperature respectively, while Γ is the reflection coefficient. All are functions of frequency in general. The sensitivity of the current *DARE* antenna falls away very quickly below 40 MHz, so instead we assume that $|\Gamma|^2 = 0.95$, roughly the value it has at 40 MHz in the current *DARE* antenna model, and does not depend on frequency. We take $T_{\mathrm{rcv}} = 50$ K. $T_{\mathrm{sky}} = T_{\mathrm{sig}} + T_{\mathrm{FG}}$, the signal plus the foregrounds, where we use the fiducial signal model of Fig. 1, and foregrounds modelled by a third-order polynomial in $\log \nu$,

$$\log T_{\mathrm{FG}} = \log T_0 + a_1 \log(\nu/\nu_0) + a_2[\log(\nu/\nu_0)]^2 + a_3[\log(\nu/\nu_0)]^3 \ , \tag{2}$$

where $\nu_0 = 100$ MHz, $T_0 = 875.0$ K, $a_1 = -2.47$, $a_2 = -0.089$ and $a_3 = 0.0127$.

The RMS noise on a spectral channel of bandwidth B is then

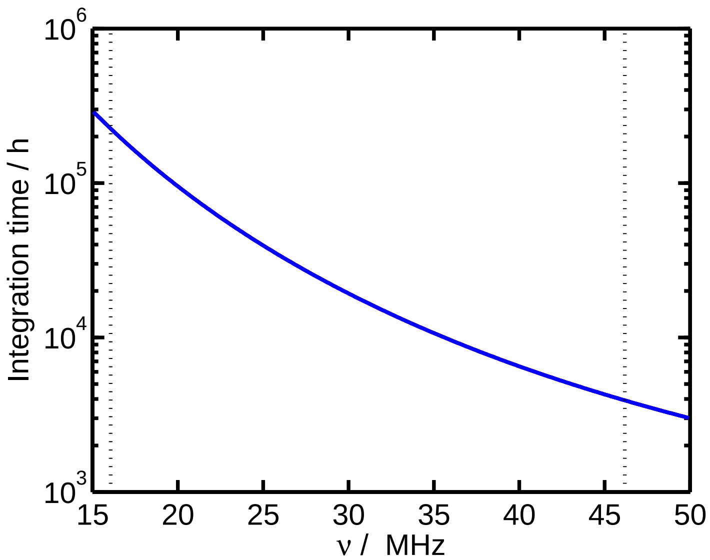
$$\sigma = \frac{T_{\mathrm{ant}}}{\sqrt{2B\tau}} \ , \tag{3}$$

for an integration time τ.

Pritchard and Loeb (2010) found that noise of the order of 1 mK allowed useful constraints on the turning points. The required integration time to reach this level of noise was also found by Harker et al. (2011) to give errors of a similar order of magnitude in a MCMC analysis, though the constraints were degraded somewhat by the requirement to fit more parameters. We will therefore adopt an RMS noise of 1 mK on a 2 MHz spectral channel as being the nominal level we need for good constraints. The 1 mK assumed by Pritchard & Loeb was on the undiminished sky temperature, T_{sky}. If $|\Gamma|$ is known, this can be inferred from T_{ant} through Eqn. (1). So, rearranging Eqn. (3),

$$\tau = \left(\frac{T_{\mathrm{sky}}}{1 \text{ mK}}\right)^2 \frac{1}{2B} = \left(\frac{T_{\mathrm{ant}}}{(1-|\Gamma|^2)\cdot 1 \text{ mK}}\right)^2 \frac{1}{2B} \ . \tag{4}$$

For the constant values we have adopted for $|\Gamma|$ and B, this just reduces to $\tau = 10^{-4}(T_{\mathrm{ant}}/1 \text{ mK})$ s, which is plotted as a function of frequency (and concentrating only on low frequencies) in Fig. 2. Near the frequency of turning point B (~ 46.2 MHz), we see that around 3000 h of integration time is



ν/MHz	τ/h
50	3000
45	4280
40	6510
35	1.07×10^4
30	1.93×10^4
25	3.95×10^4
20	9.54×10^4
15	2.94×10^5

Figure 2: Integration time in hours to reach a noise level of 1 mK on the inferred sky spectrum. Dotted lines show the fiducial frequency of turning points A and B. Some representative values are also tabulated.

required, which accords roughly with how much time was required to constrain turning point B in the MCMC analysis. Note that above 40 MHz the *DARE* reflection coefficient used in the MCMC analysis is smaller than the constant reflection coefficient used here, making the instrument more sensitive and leading to a smaller integration time. This explains the slightly larger τ values seen here compared to

Harker et al. (2011). The time needed increases a great deal at low frequencies, so to probe these would require either a more sensitive system, multiple antennas, lower receiver noise, or some combination of all of these.

3 Fisher matrix analysis

The noise level is only an indirect measure of what we would really like to know, which is how well the turning points can be constrained given a certain instrumental bandwidth and integration time. We have carried out a Fisher matrix analysis akin to that described by Pritchard and Loeb (2010) to compute the constraints on the turning point parameters for different scenarios.

A Fisher matrix analysis is not a foreground subtraction technique working with a specific realization of a (real or simulated) dataset like the MCMC code we developed. Rather, it is a way of finding a lower bound on the error which can be attained on the parameters of interest in a general case. It is much faster and can be used to quickly explore a wide range of different models for the signal, foregrounds, bandwidth, etc.

The Fisher matrix (or Fisher *information* matrix) represents the amount of information that a dataset contains about a set of parameters. Under some assumptions about the noise and the form of the likelihood surface, and assuming a simple form for the instrumental response, the Fisher matrix for this problem can be written as

$$F_{ij} = \sum_{n=1}^{N_{\mathrm{channel}}} (2 + B\tau) \frac{\mathrm{d}\log T_{\mathrm{sky}}(\nu_n)}{\mathrm{d}p_i} \frac{\mathrm{d}\log T_{\mathrm{sky}}(\nu_n)}{\mathrm{d}p_j} \tag{5}$$

where T_{sky} is a function of the parameters $\{p_i\}$ of the foreground and signal models (see Pritchard and Loeb, 2010, and references therein). If T_{sky} varies rapidly as parameter p_i is changed, then clearly the value of T_{sky} contains a lot of information about parameter p_i (which is reflected in a large Fisher matrix element), and a measurement of T_{sky} will allow p_i to be constrained with small error. $|F_{ij}|$ also increases as τ increases, because increasing the integration time increases the amount of information we have about the parameters.

The intuition that if T_{sky} varies rapidly as p_i is varied then we can achieve tight constraints on p_i is formalised by the Cramér-Rao bound, which states that the error, σ_i, which can be achieved on the parameter p_i is bounded such that $\sigma_i \geq \sqrt{(F^{-1})_{ii}}$. If we can compute the inverse of the Fisher matrix, this tells us the best errors we could expect to achieve on the parameters of our model.

We will concentrate on the errors which can be achieved on the frequency of the turning points as a function of the observed frequency band for a *DARE*-like experiment. The frequency band is determined by two parameters, its minimum frequency ν_{min} and its maximum frequency ν_{max}. Clearly, our constraints will always improve if ν_{min} is decreased or ν_{max} is increased. Unfortunately, this isn't always possible, so the aim of this analysis is to look for an optimal combination of ν_{min} and ν_{max} within the constraints imposed by having to build a system that can actually achieve them, and to see how the errors on the turning point frequencies could be improved with a hypothetical instrument that could observe extremely low frequencies (to test if it would be worth trying to develop one at this stage).

Fig. 3 shows how the errors on the turning points depend on the observing band for 10 000 h of observing time; each panel corresponds to one of the four turning points. Each point in one of the plots represents a different choice for the observing band, and the contours show lines of equal error in this space. We can move to regions with lower error by moving further left or up in this space. However, given a particular antenna design and a particular choice of ν_{min}, the largest frequency the antenna can usefully probe may be given by $\nu_{\mathrm{max}} = k\nu_{\mathrm{min}}$ for some k. The *DARE* antenna concept seems to yield $k \approx 3$. The three diagonal lines in the panel for turning point C (lower left) are lines of constant k, for $k = 2, 3, 4$. The fiducial *DARE* band of 40–120 MHz, marked with a '+', lies on the middle of these three lines, and one can imagine scaling the antenna to move it along that line. One can move along that line to try to optimize the band without having to fundamentally change the antenna design.

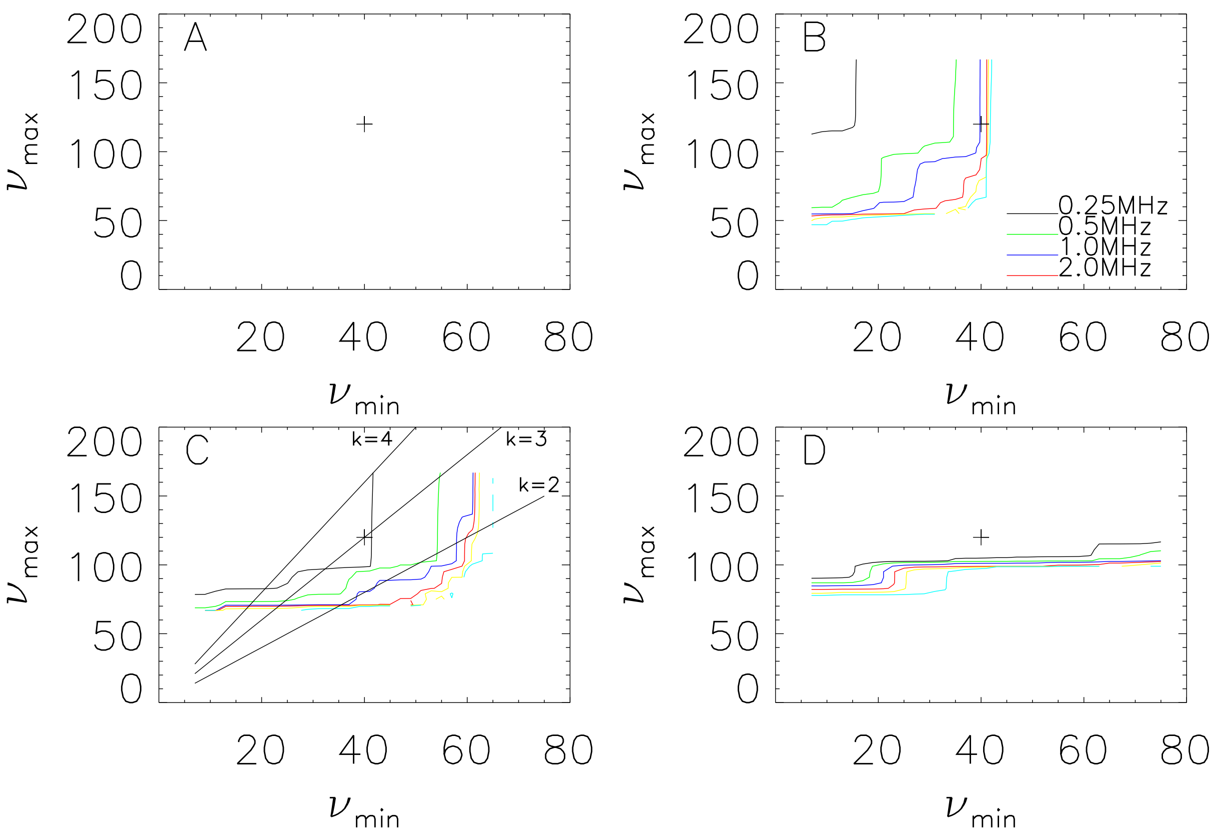


Figure 3: The panels show, in reading order, contour plots of the error on the frequency of turning point A, B, C and D respectively, for 10 000 h of integration time, as a function of $\nu_{\rm min}$ and $\nu_{\rm max}$. The black contour, furthest to the upper left, is an error of 0.25 MHz; subsequent contours (green, blue, red, yellow, cyan) successively increase the error by a factor of two. The straight black lines in the lower left panel (turning point C) show $\nu_{\rm max} = k\nu_{\rm min}$ for $k = 2, 3, 4$. The fiducial *DARE* band of 40–120 MHz is shown in each panel with a '+'. There are no contours in the upper left panel because turning point A cannot be constrained to within 8 MHz for 10 000 h of integration no matter what $\nu_{\rm min}$ is.

In fact the current *DARE* band appears to be somewhere near the optimum, though arguably it could be shifted to slightly lower frequencies to improve recovery of turning points B and C, without increasing the error on turning point D to more than 0.25 MHz. The frequency of turning point D is of course uncertain, however, and so this would risk missing out. A test of how likely this might be would require us to look at a wider range of signal models.

The largest contour in each panel is for an error of 8 MHz. No such contour appears in the upper left panel of Fig. 3, indicating that 10 000 h of integration is insufficient to constrain turning point A to within 8 MHz no matter what band is chosen.

If we do not aim to constrain turning point A, and instead consider its position fixed, the constraints on the other turning points can be improved (we use the information in the spectrum to constrain the other turning points rather than A, at the risk of biasing our estimates). The results of doing so are shown in Fig. 4, which assumes an integration time of 4000 h. The constraints on turning point B for the fiducial *DARE* band improve even though the amount of integration time has gone down, but the other conclusions are largely unaffected.

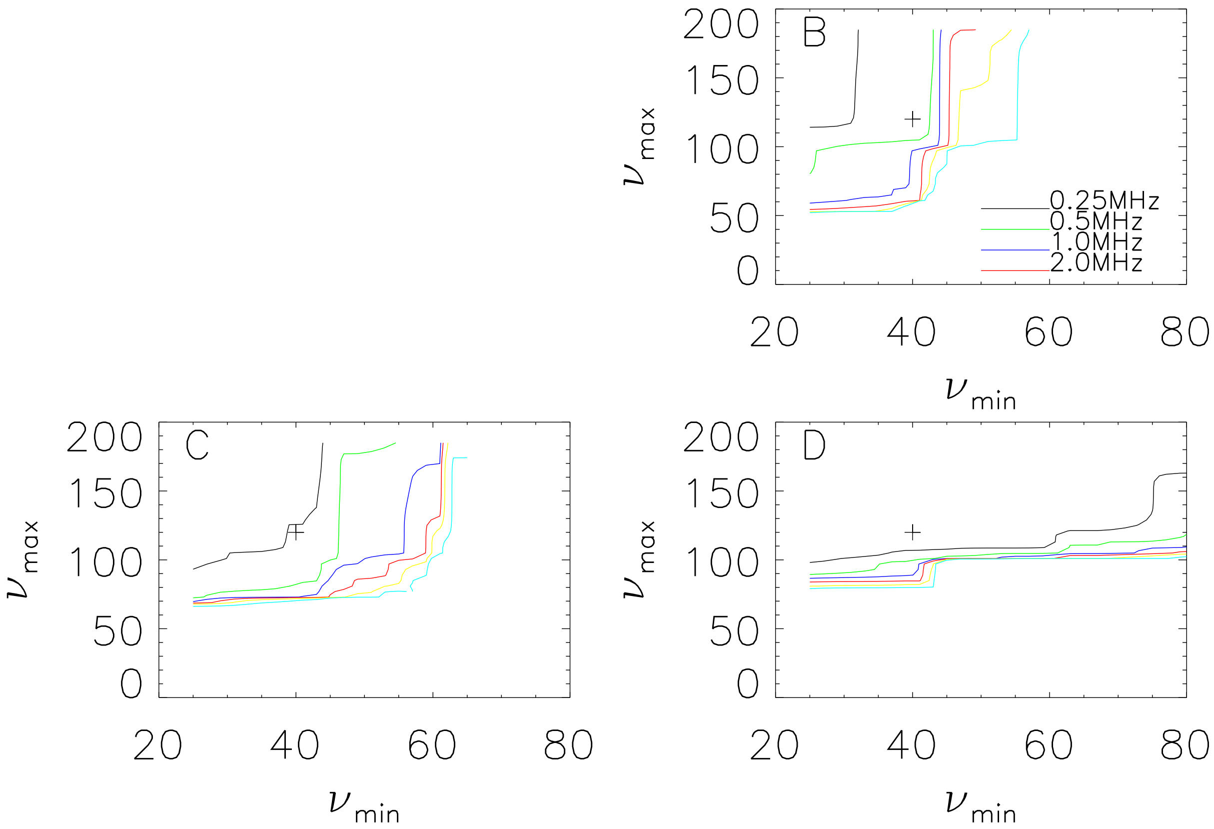


Figure 4: As Fig. 3, except that the position of turning point A is fixed so constraints are not shown, and only 4000 h of integration is assumed.

4 Conclusions

We have looked at the effect of changing the frequency coverage of a *DARE*-like system on the errors on a model of the highly redshifted 21-cm spectrum. We started by carrying out a simple computation of the integration time required to reach a noise level of 1 mK on a 2 MHz spectral channel, before moving on to a more sophisticated Fisher matrix analysis of the constraints on the turning points of the signal.

The Fisher matrix analysis is more model-dependent: the errors inferred on the turning points depend on their 'true' position in the fiducial model. Assuming this model, the analysis showed that a small improvement in the errors on the position of turning point B could be achieved if the bottom end of the frequency band was lowered to ~ 35 MHz. If a frequency coverage of a factor of ~ 3 is possible, however, this may result in our missing out entirely on turning point D (though we would in fact detect it if our fiducial signal model is correct). The current *DARE* band of 40–120 MHz is none the less close to optimal, and our MCMC analysis suggests it is adequate for a detection of turning point B within 2000 h if our models are accurate (Harker et al., 2011). Other models were considered by Pritchard and Loeb (2010), who found that 40–120 MHz performed reasonably well for turning point parametrizations of 21-cm histories produced using a wide range of astrophysical parameters.

A more convincing motivation to go to lower frequency is therefore to probe the real 'Dark Ages', before the first stars produce Lyα and cause turning point B. Our simple noise computation of Section 2 suggests, however, that this will be very difficult. The size of the foregrounds increases very rapidly at low frequency, pushing the required integration time far past what can be achieved within the lifetime of a satellite mission with a single antenna. This conclusion is supported by the Fisher matrix analysis, which suggests 10 000 h of integration does not allow us to locate turning point A to within 8 MHz with a single antenna whatever frequency band is assumed. The position of turning point A is rather robust to variations in astrophysical parameters, so we would not expect our conclusions about the difficulty of detecting it to change.

References

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